Age-and field-size-parameterized calculations of physiologically significant XYZ colour-matching functions

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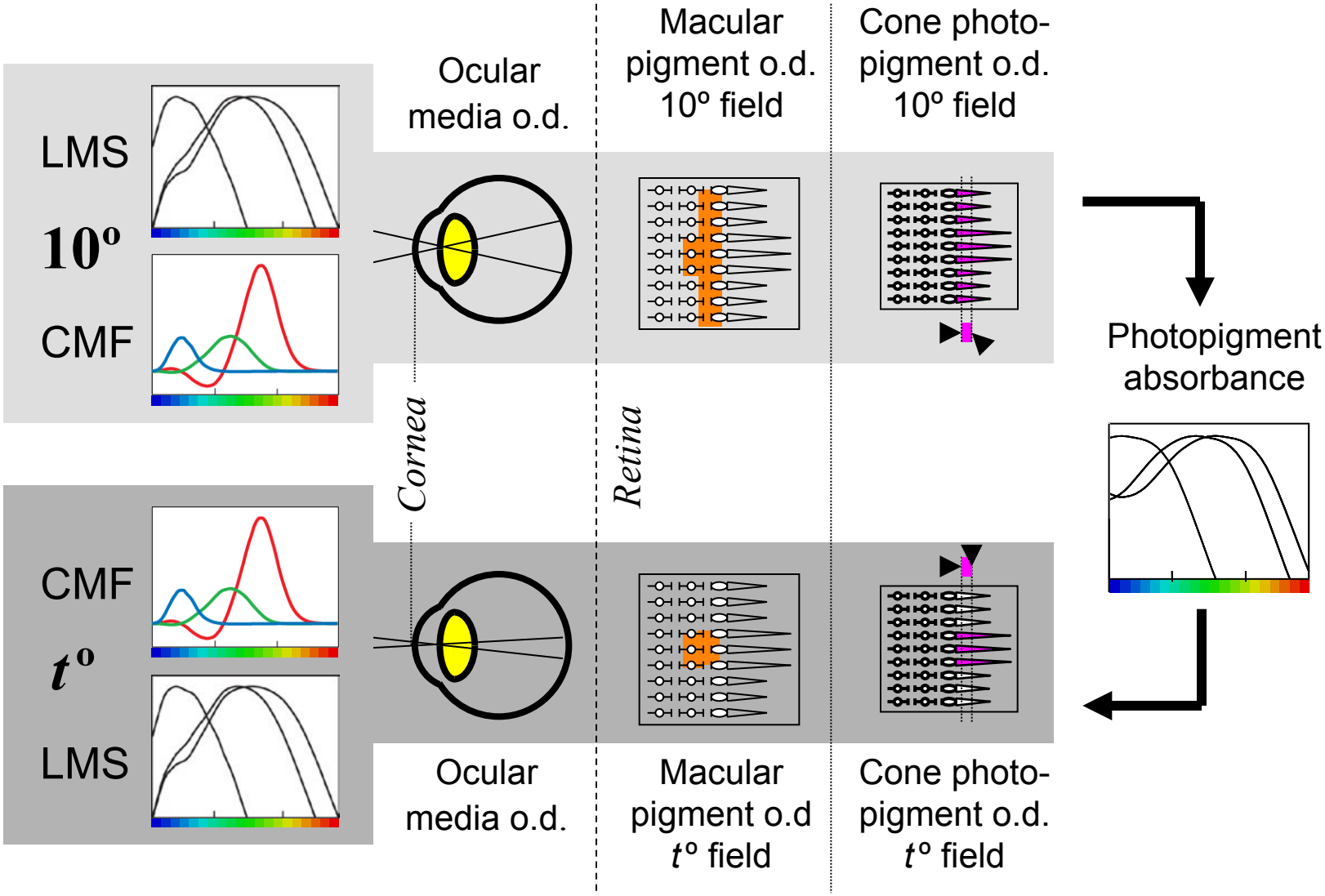
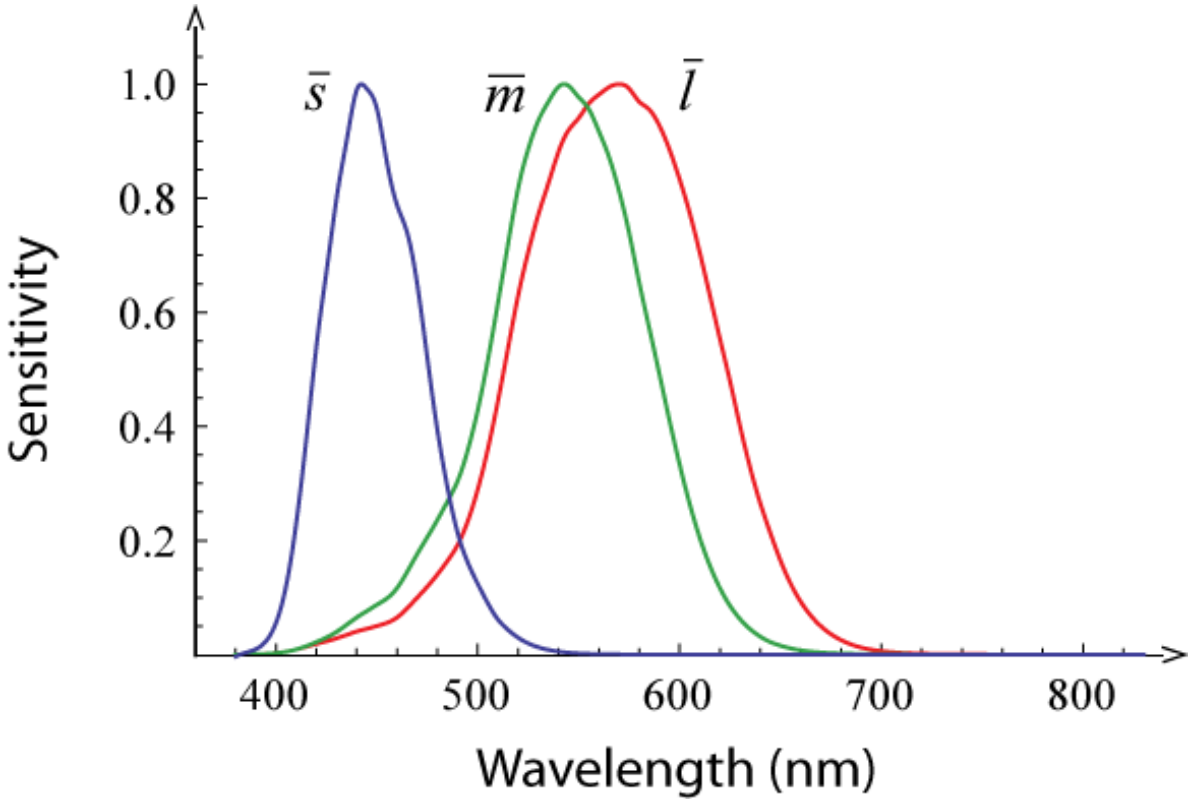
Reconstructing Fundamentals (*t* º field)



Cone fundamentals

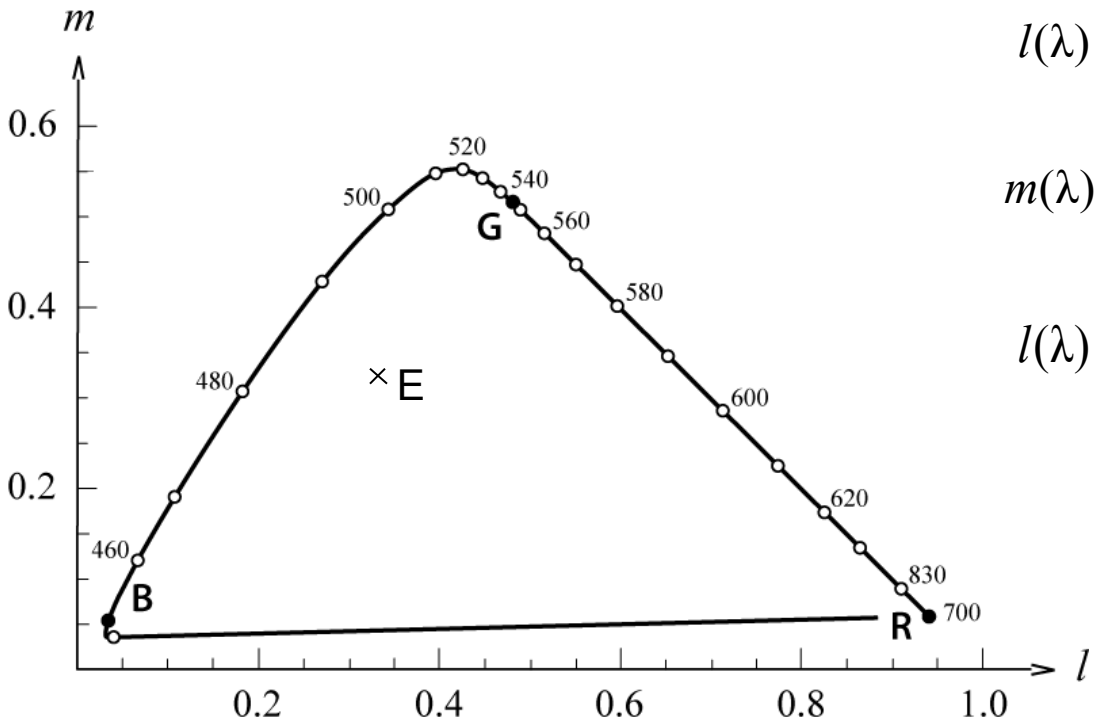
$\bar{l}(\lambda),\ \bar{m}(\lambda),\ \bar{s}(\lambda)$: Stockman-Sharpe 2° cone fundamental

[*Stockman, Sharpe and Fach* (*1999, 2000*)]



Maxwellian (l, m) diagram

$\bar{l}(\lambda),\ \bar{m}(\lambda),\ \bar{s}(\lambda)$: CIE 2006 $t°$ cone fundamentals (energy units)



$$l(\lambda) = \frac{\bar{l}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

$$m(\lambda) = \frac{\bar{m}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

$$l(\lambda) = \frac{\bar{s}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

λ_R = 700.0 nm

λ_G = 546,1 nm

λ_B = 435.8 nm

Assumptions

Colorimetry

1) Grassmann's laws of additive colour mixture hold true.

Physiology

2) The sensation of a colour stimulus originates from the light absorption of three different photosensitive pigments contained in the cones.

3) For all photoreceptors containing a particular pigment the relative spectral absorption factor of that pigment (relative to the radiant power incident on the cone) is the same and remains unchanged whatever stimuli are applied.

4) *The Rushton principle of univariance* :
Once light is absorbed in the cones, the only remaining information is the photon (quantum) count in each cone (not the wavelength of the absorbed photons).

Linear mapping

between cone exitations and tristimulus values

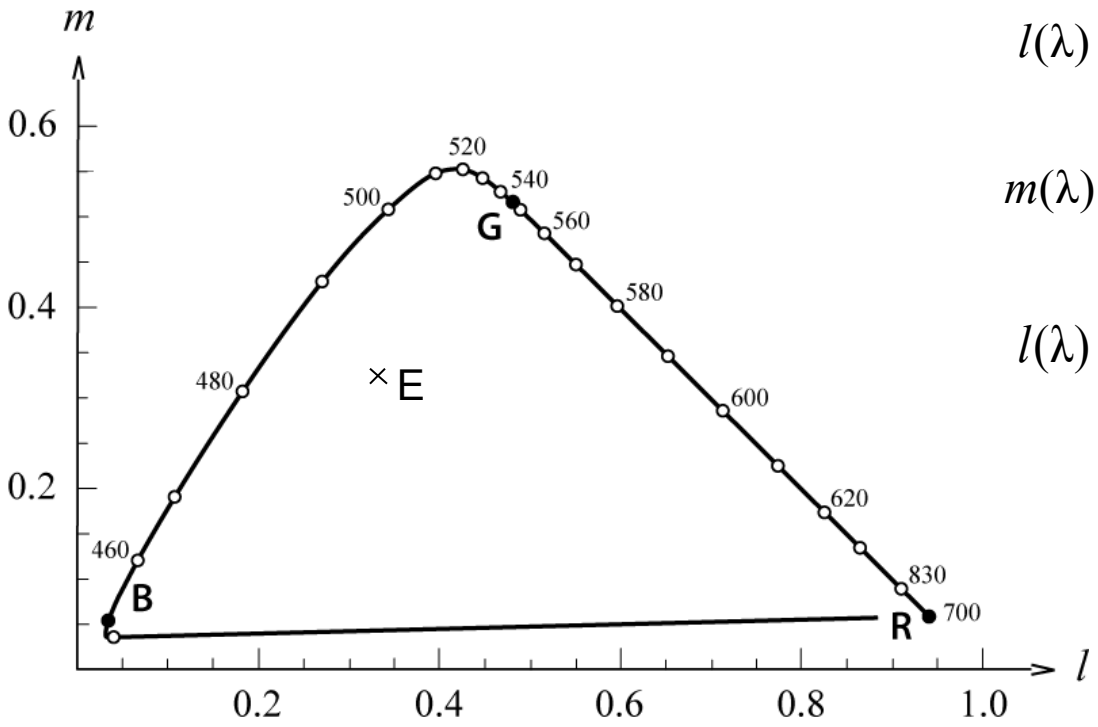
$$\begin{pmatrix} R \\ G \\ B \end{pmatrix}_{\text{F, fs, age}} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}_{\text{fs, age}} \begin{pmatrix} L \\ M \\ S \end{pmatrix}_{\text{fs, age}}$$

↓ Colorimetrically based vector space

↓ Physiologically based vector space

Maxwellian (l, m) diagram

$\bar{l}(\lambda),\ \bar{m}(\lambda),\ \bar{s}(\lambda)$: CIE 2006 $t°$ cone fundamentals (energy units)



$$l(\lambda) = \frac{\bar{l}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

$$m(\lambda) = \frac{\bar{m}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

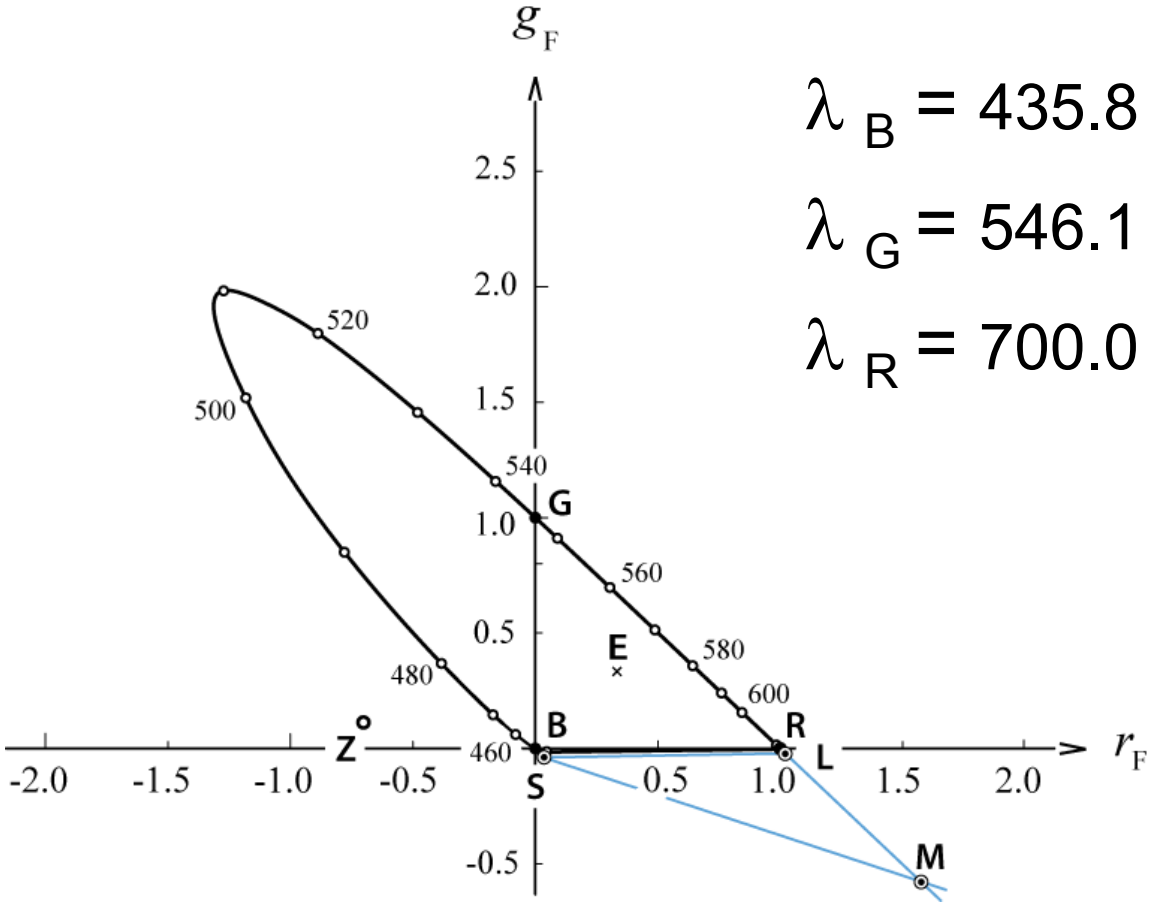
$$l(\lambda) = \frac{\bar{s}(\lambda)}{\bar{l}(\lambda) + \bar{m}(\lambda) + \bar{s}(\lambda)}$$

λ_R = 700.0 nm

λ_G = 546,1 nm

λ_B = 435.8 nm

(r_F, g_F) chromaticity diagram



λ_B = 435.8 nm

λ_G = 546.1 nm

λ_R = 700.0 nm

XYZ concept

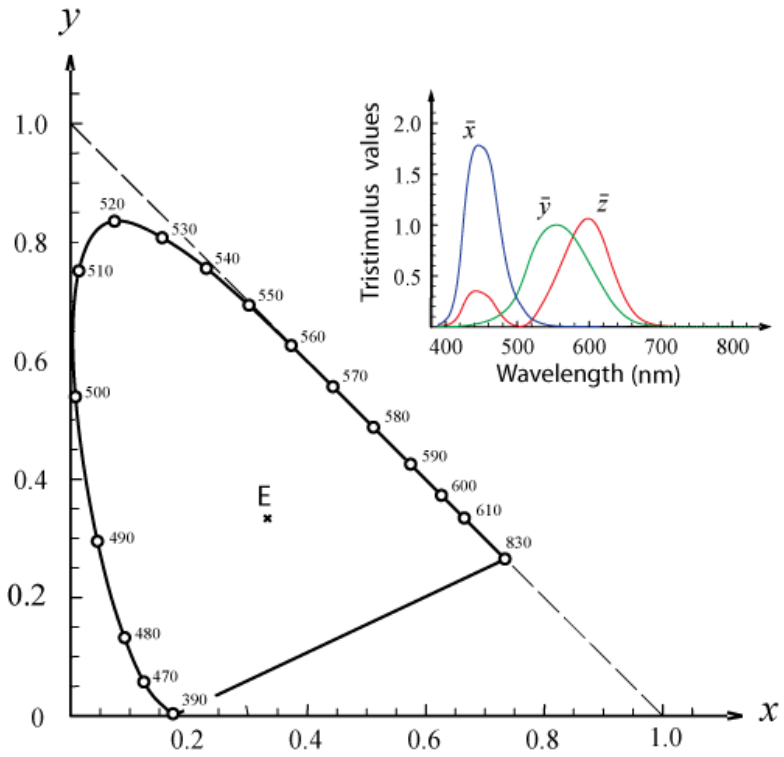
1) All tristimulus values of any (real) colour stimulus are to be nonnegative.

2) The luminosity coefficients (i.e., associated luminous fluxes) of the X-, the Y-, and the Z-primary are to be 0, 1, and 0, respectively (when determined relative to the spectral luminous efficiency function representative of the viewing angle).

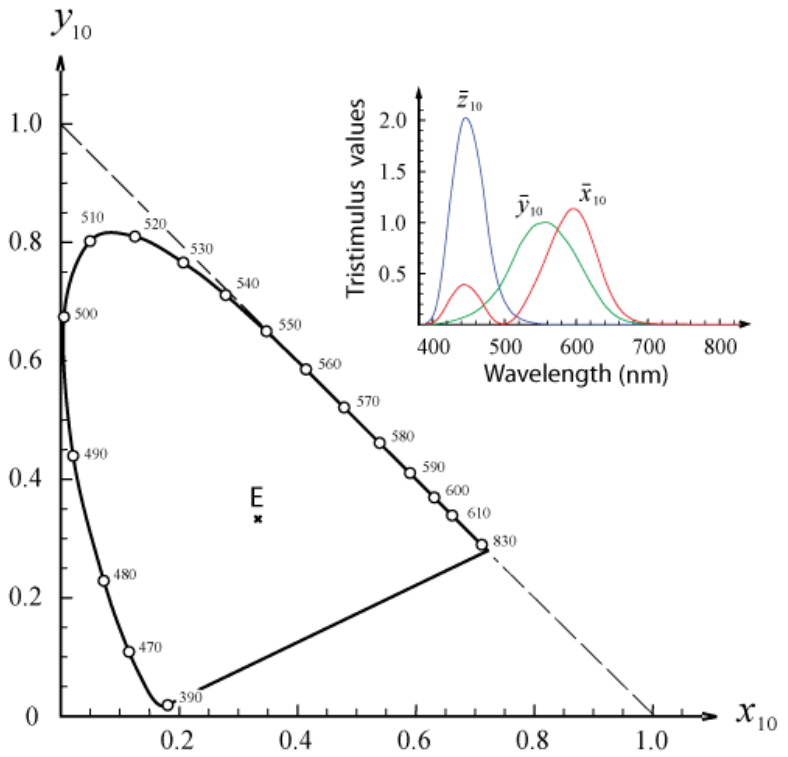
3) The tristimulus values of Illuminant E (the equi-energy spectrum) are to be equal.

CIE XYZ colorimetric systems

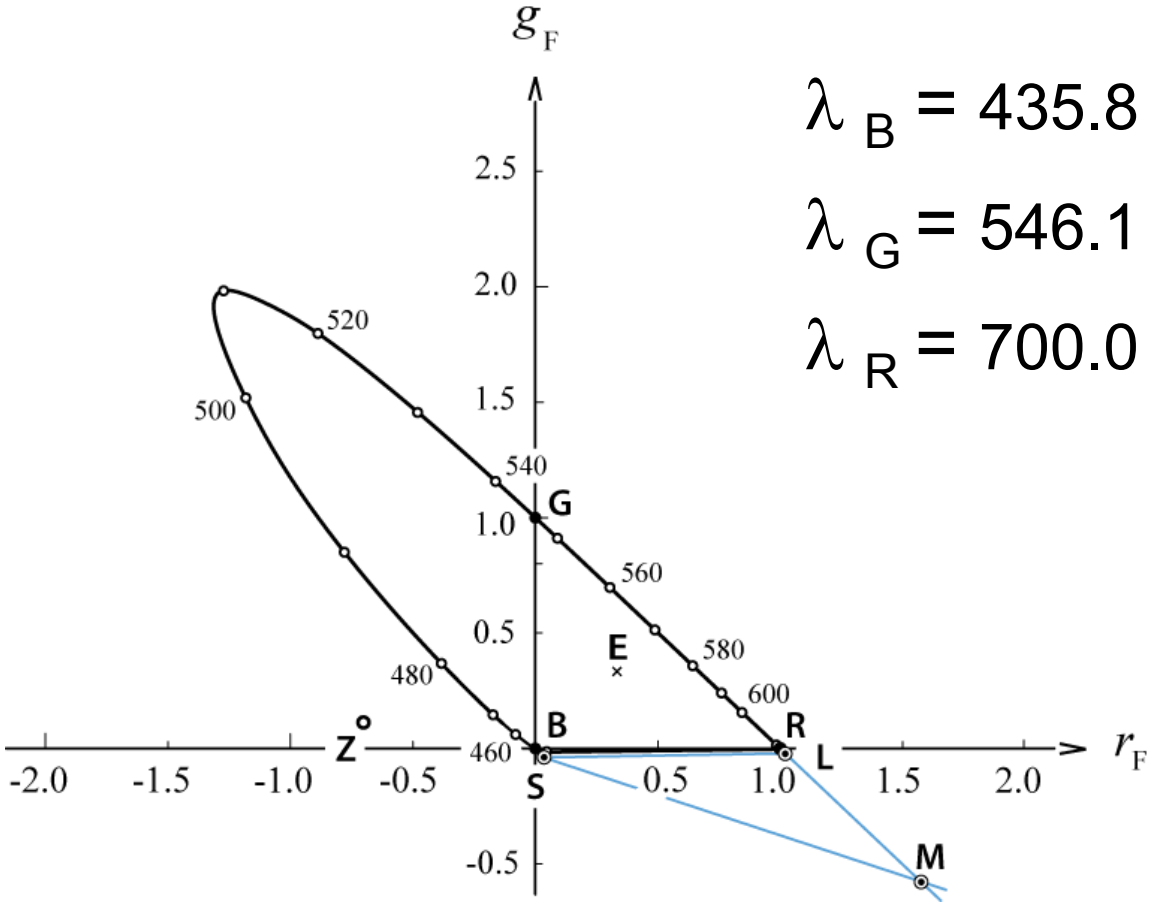
CIE $_{1931}$ standard colorimetric system (X, Y, Z)



CIE $_{1964}$ standard colorimetric system (X_{10} , Y_{10} , Z_{10})



(r_F, g_F) chromaticity diagram



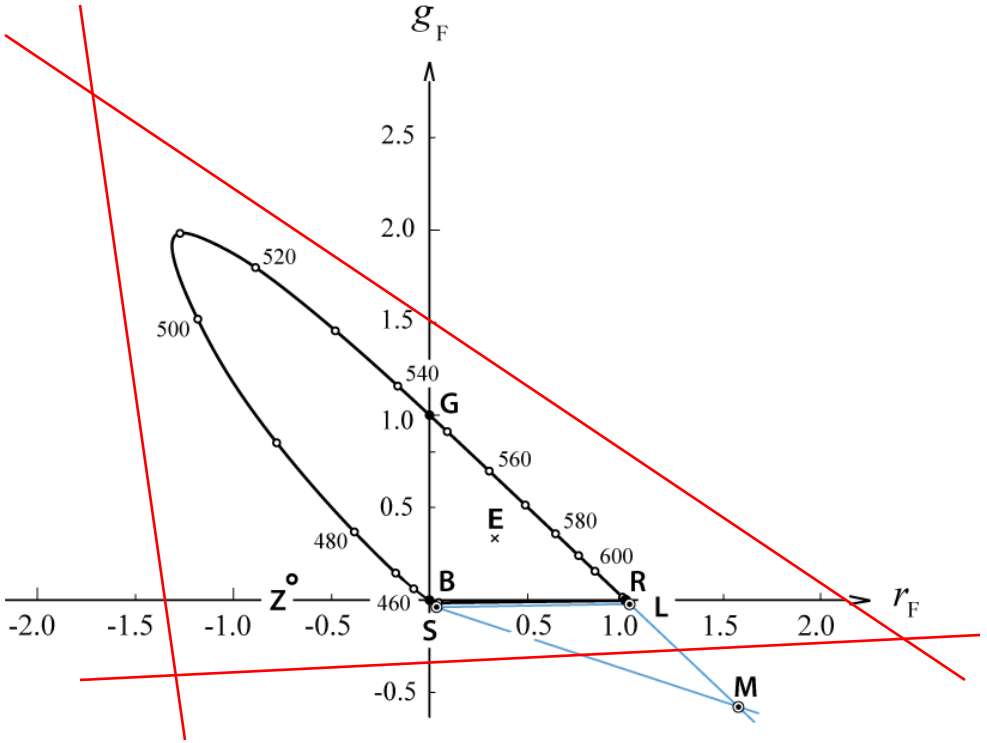
λ_B = 435.8 nm

λ_G = 546.1 nm

λ_R = 700.0 nm

Criterion 1: All tristimulus values of any (real) colour stimulus are to be nonnegative.

Circumscription of spectrum locus



Peak optical densities of the L- and M-cone photopigments

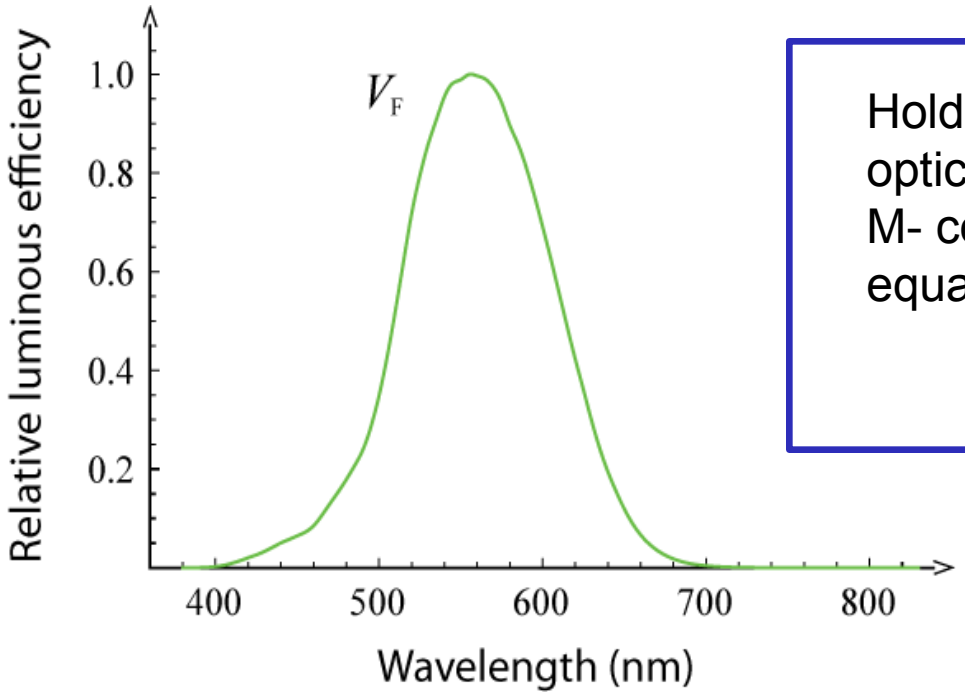
Field size (º)	D(L- and M- pigment)	D(S-pigment)
1	0,635	0,513
2	0,500	0,400
3	0,437	0,347
4	0,407	0,322
5	0,393	0,311
6	0,386	0,305
7	0,383	0,302
8	0,381	0,301
9	0,381	0,301
10	0,380	0,300

$$V_{F,\,fs,\,age,\,q}(\lambda) = \frac{1,89\,\overline{l}_{fs,\,age,\,q}(\lambda) + \overline{m}_{fs,\,age,\,q}(\lambda)}{[\,1,89\,\overline{l}_{fs,\,age,\,q}(\lambda) + \overline{m}_{fs,\,age,\,q}(\lambda)\,]_{max}}$$

Criterion 2:

$$\bar{y}_{F,fs,age}(\lambda) \equiv V_{F,fs,age}(\lambda)$$

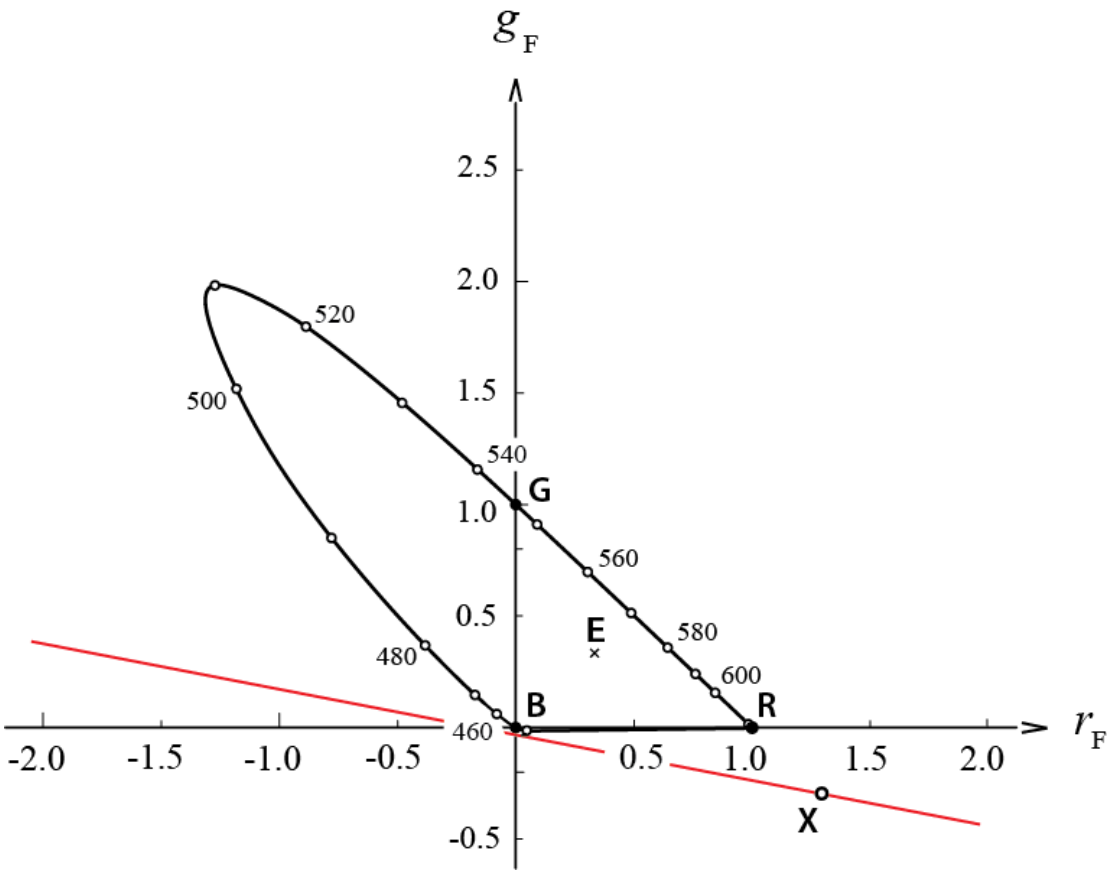
$$\bar{y}_{F,fieldsize,age,q}(\lambda) = \frac{1,89\ \bar{l}_{fs,age,q}(\lambda) + \bar{m}_{fs,age,q}(\lambda)}{[\,1,89\ \bar{l}_{fs,age,q}(\lambda) + \bar{m}_{fs,age,q}(\lambda)\,]_{max}}$$



Holds true provided the peak optical densities of the L- and M- cone photo-pigments are equal

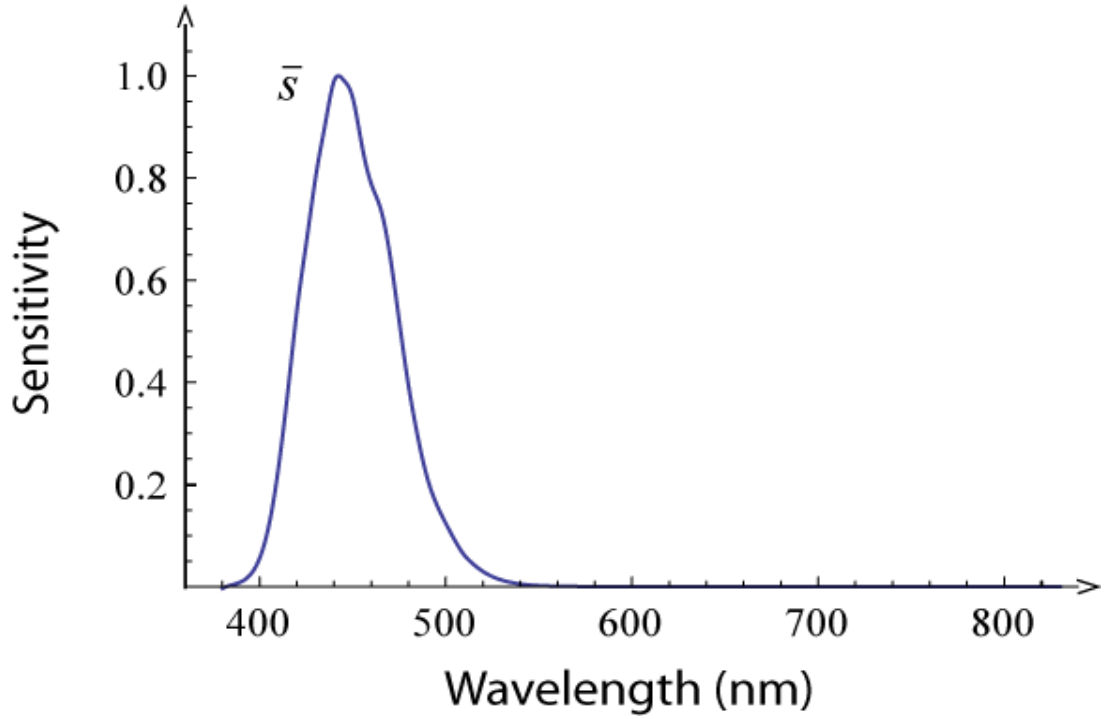
$$D_{\tau,max,L} = D_{\tau,max,M}$$

Circumscribing line 1



Criterion 3: $\overline{z}_{\mathrm{F}}(\lambda) \propto \overline{s}(\lambda)$

$$\overline{z}_{\text{F, fieldsize, age}}(\lambda) = k_{\text{F, fieldsize, age}}\ \overline{s}_{\text{F, fieldsize, age}}(\lambda)$$



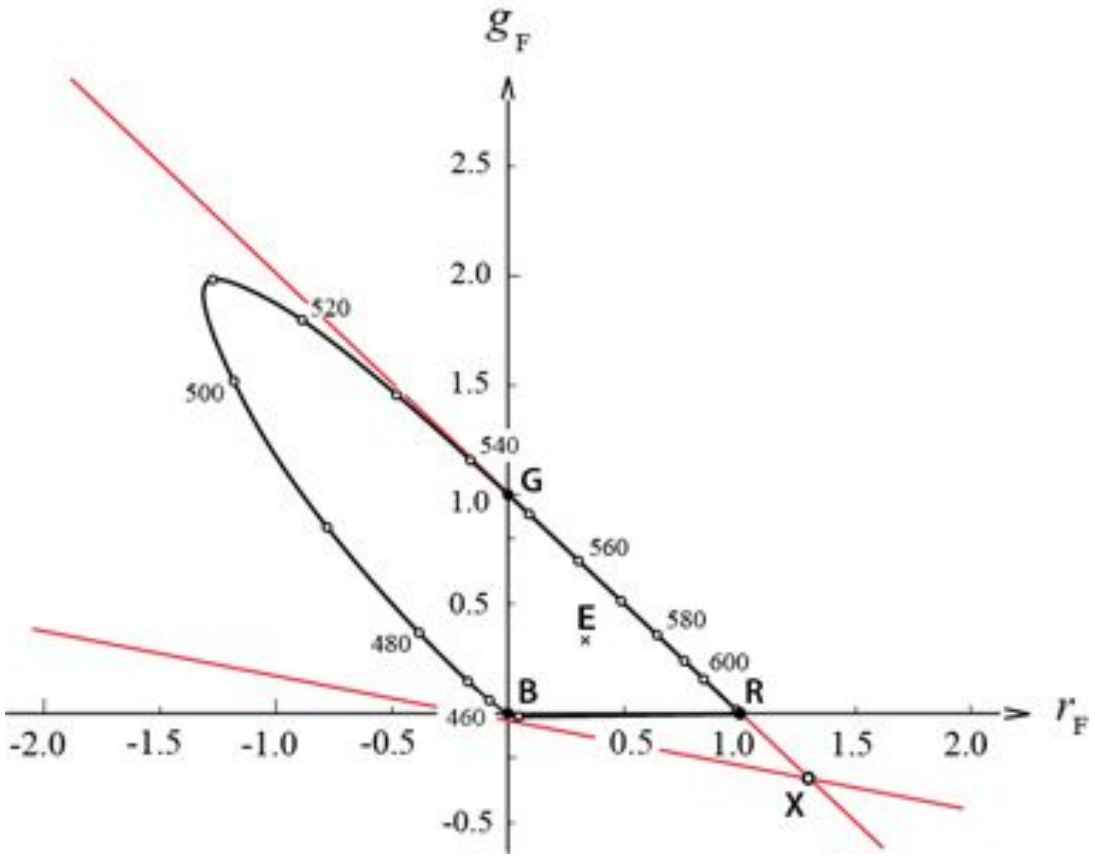
Criterion 4:

$$\sum_{\lambda} \overline{x}_{\text{F, fs, age}}(\lambda) = \sum_{\lambda} \overline{y}_{\text{F, fs, age}}(\lambda) = \sum_{\lambda} \overline{z}_{\text{F, fs, age}}(\lambda)$$

$$\overline{z}_{\text{F, fs, age}}(\lambda) = k_{\text{F, fs, age}}\ \overline{s}_{\text{F, fs, age}}(\lambda)\ ,$$

$$k_{\text{F, fs, age}} = \frac{\sum_{\lambda} V_{\text{F, fs, age}}(\lambda)}{\sum_{\lambda} \overline{s}_{\text{F, fs, age}}(\lambda)}$$

Circumscribing lines 1 and 2



So far …

$$\overline{y}_{\mathrm{F,\,fs,\,age,\,q}}(\lambda) = \frac{1{,}89\ \overline{l}_{\mathrm{fs,\,age,\,q}}(\lambda) + \overline{m}_{\mathrm{fs,\,age,\,q}}(\lambda)}{[\,1{,}89\ \overline{l}_{\mathrm{fs,\,age,\,q}}(\lambda) + \overline{m}_{\mathrm{fs,\,age,\,q}}(\lambda)\,]_{\max}}$$

$$\overline{z}_{\mathrm{F,\,fs,\,age}}(\lambda) = k_{\mathrm{F,\,fs,\,age}}\ \overline{s}_{\mathrm{F,\,fs,\,age}}(\lambda)\ ,$$

$$k_{\mathrm{F,\,fs,\,age}} = \frac{\sum_{\lambda} V_{\mathrm{F,\,fs,\,age}}(\lambda)}{\sum_{\lambda} \overline{s}_{\mathrm{F,\,fs,\,age}}(\lambda)}$$

So far …

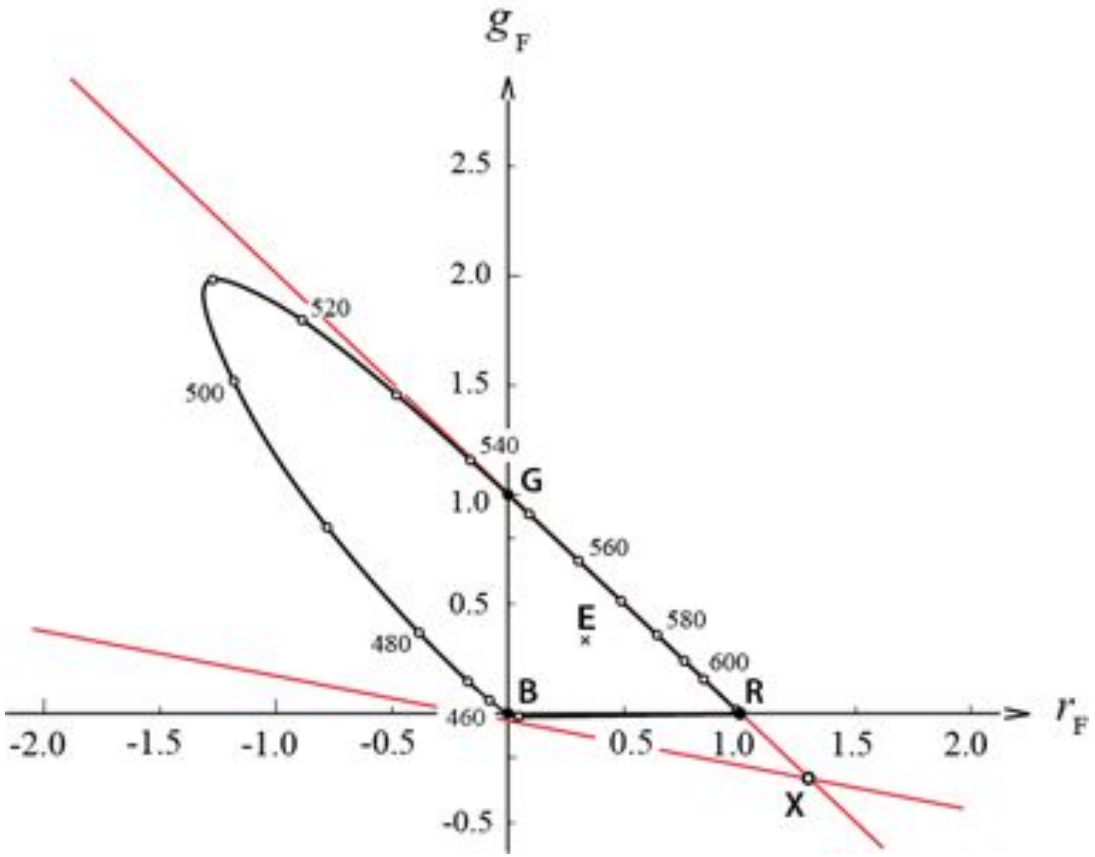
$$\overline{x}_{\text{F, fs, age}}(\lambda) = ?$$

$$\overline{y}_{\text{F, fs, age, q}}(\lambda) = \frac{1,89\ \overline{l}_{\text{fs, age, q}}(\lambda) + \overline{m}_{\text{fs, age, q}}(\lambda)}{[\,1,89\ \overline{l}_{\text{fs, age, q}}(\lambda) + \overline{m}_{\text{fs, age, q}}(\lambda)\,]_{\max}}$$

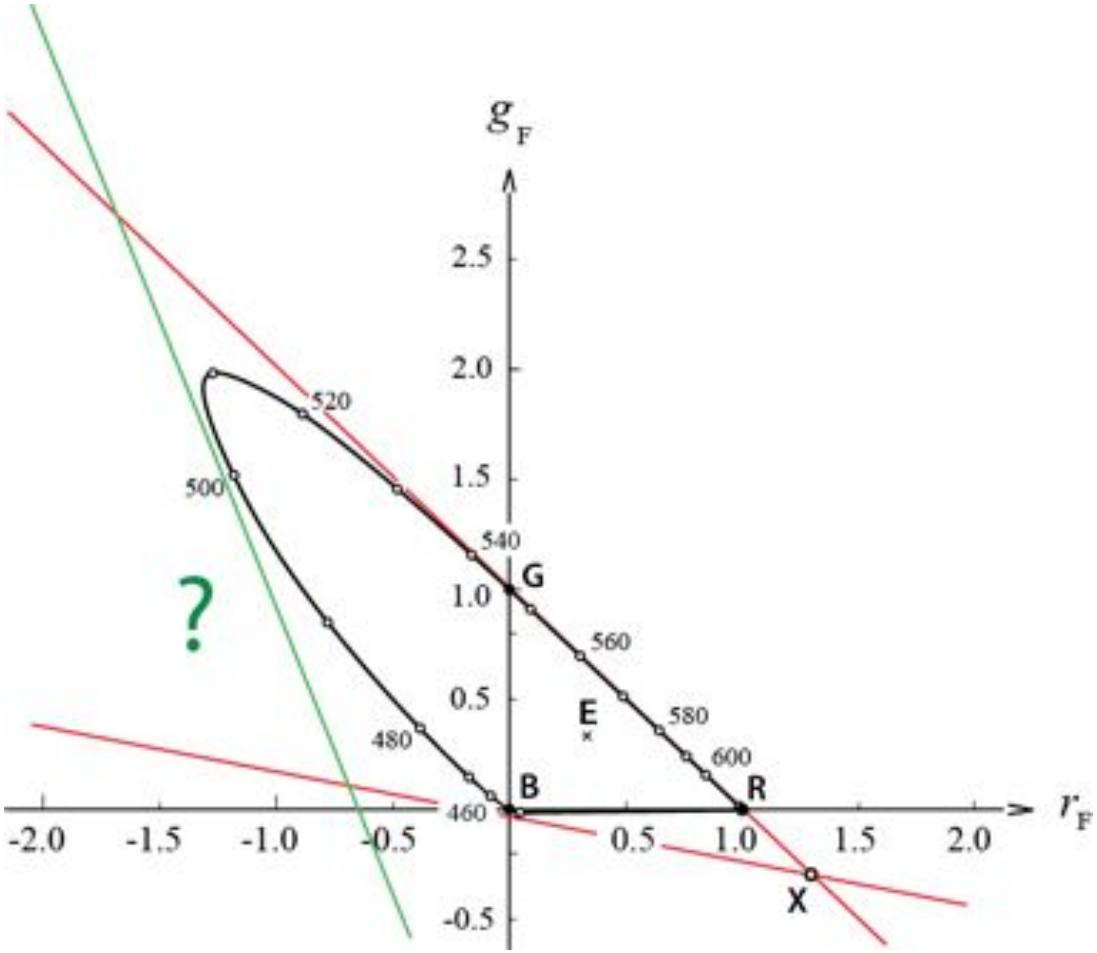
$$\overline{z}_{\text{F, fs, age}}(\lambda) = k_{\text{F, fs, age}}\ \overline{s}_{\text{F, fs, age}}(\lambda)\ ,$$

$$k_{\text{F, fs, age}} = \frac{\sum_{\lambda} V_{\text{F, fs, age}}(\lambda)}{\sum_{\lambda} \overline{s}_{\text{F, fs, age}}(\lambda)}$$

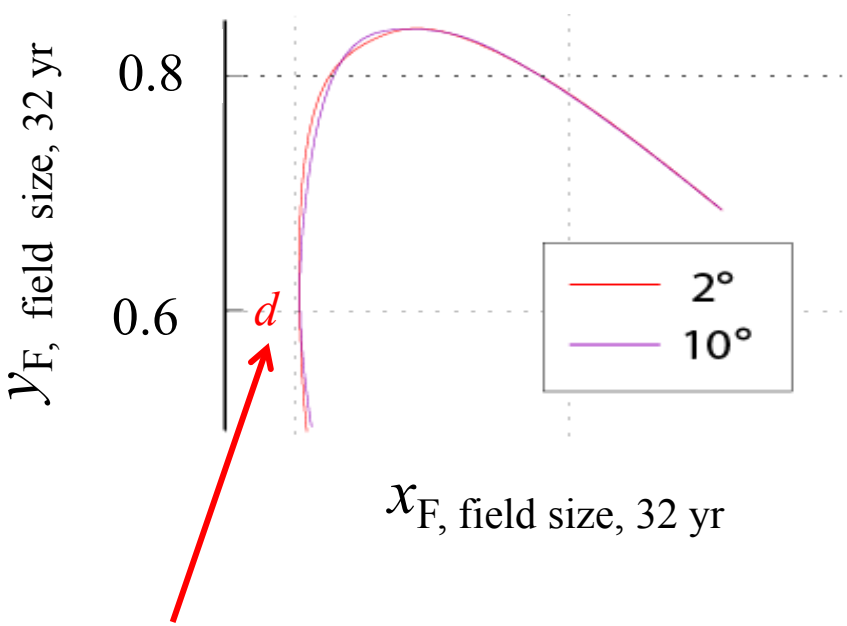
Circumscribing lines 1 and 2



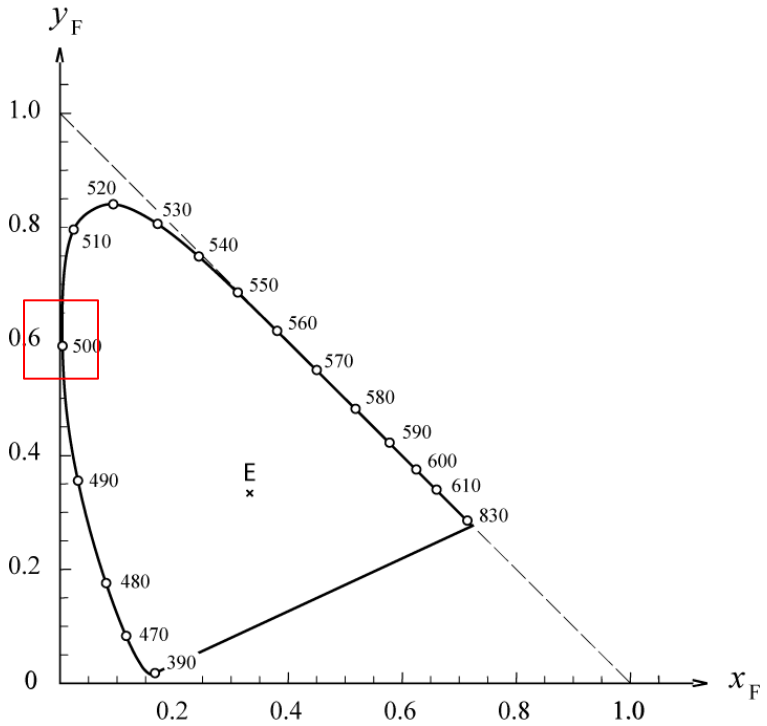
The remaining line ?



Criterion 5: $\min\left(x_{\text{F, fieldsize, age}}(\lambda)\right) = \min\left(x_{\text{ref, }\lambda}(\lambda)\right)$



$$d = \min\left(x_{\text{F, fieldsize, age}}(\lambda)\right) = \min\left(x_{\text{ref}}(\lambda)\right)$$

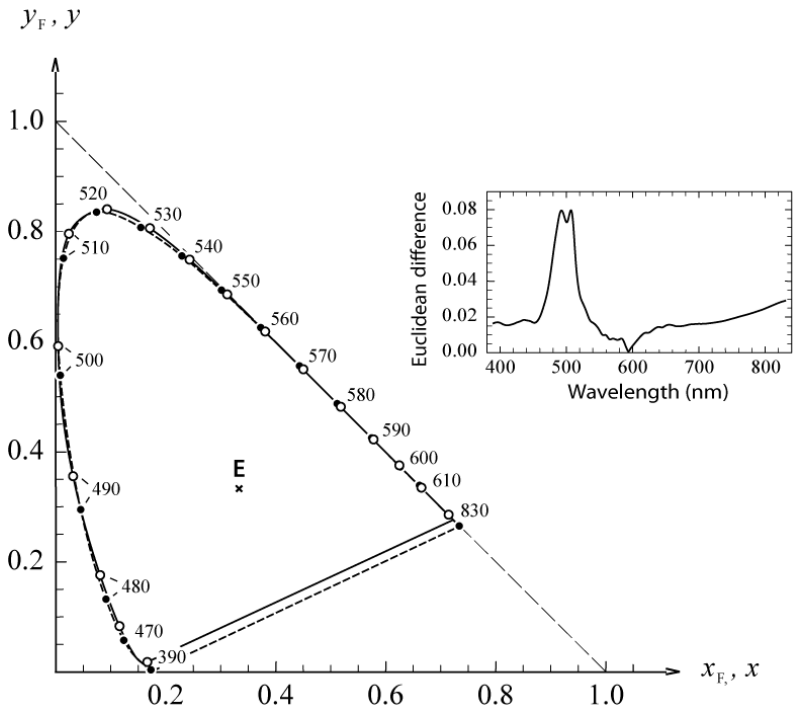


Criterion 6:

$$\Delta_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{\lambda}\left(\left(x_{\text{F},\lambda} - x_{\lambda}\right)^2 + \left(y_{\text{F},\lambda} - y_{\lambda}\right)^2 + \left(z_{\text{F},\lambda} - z_{\lambda}\right)^2\right)} = \min\left(\Delta_{\text{rms}}\right)$$

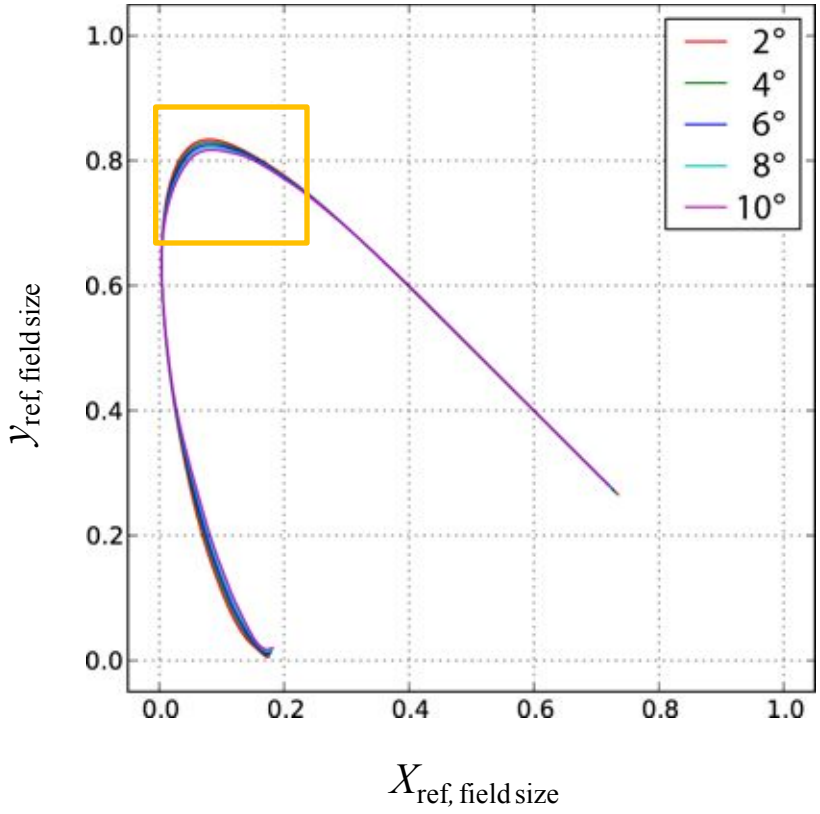
… together with Criterion 4 :

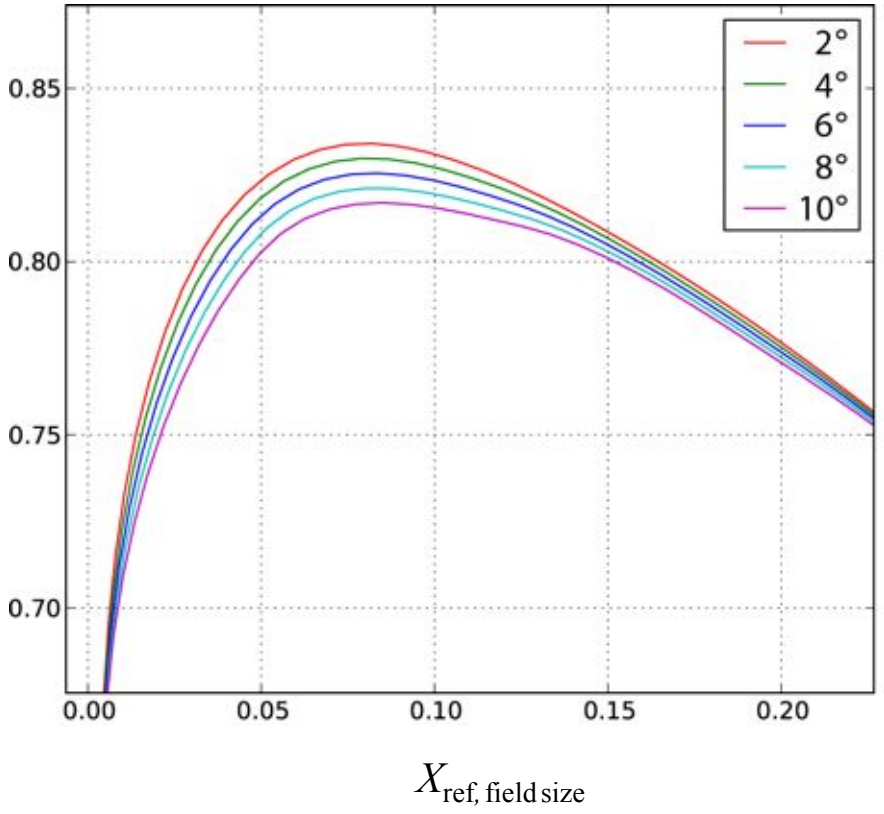
$$\sum_{\lambda} \overline{x}_{\text{F, fs, age}}(\lambda) = \sum_{\lambda} \overline{y}_{\text{F, fs, age}}(\lambda)$$



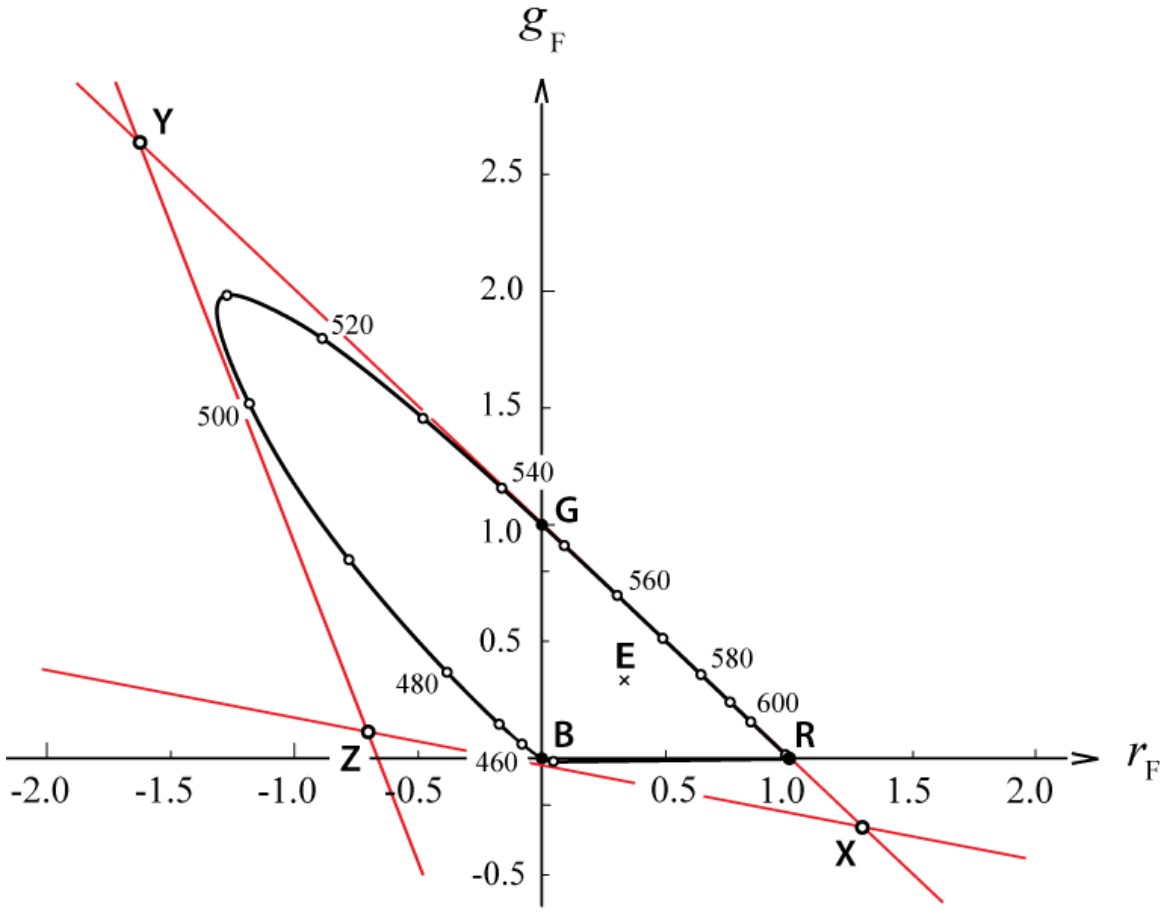
Reference Diagrams

(Morphed with Respect to Field Size)





Circumscribing lines 1, 2 and 3



Transformation LMS → XYZ

from cone exitations to XYZ tristimulus values

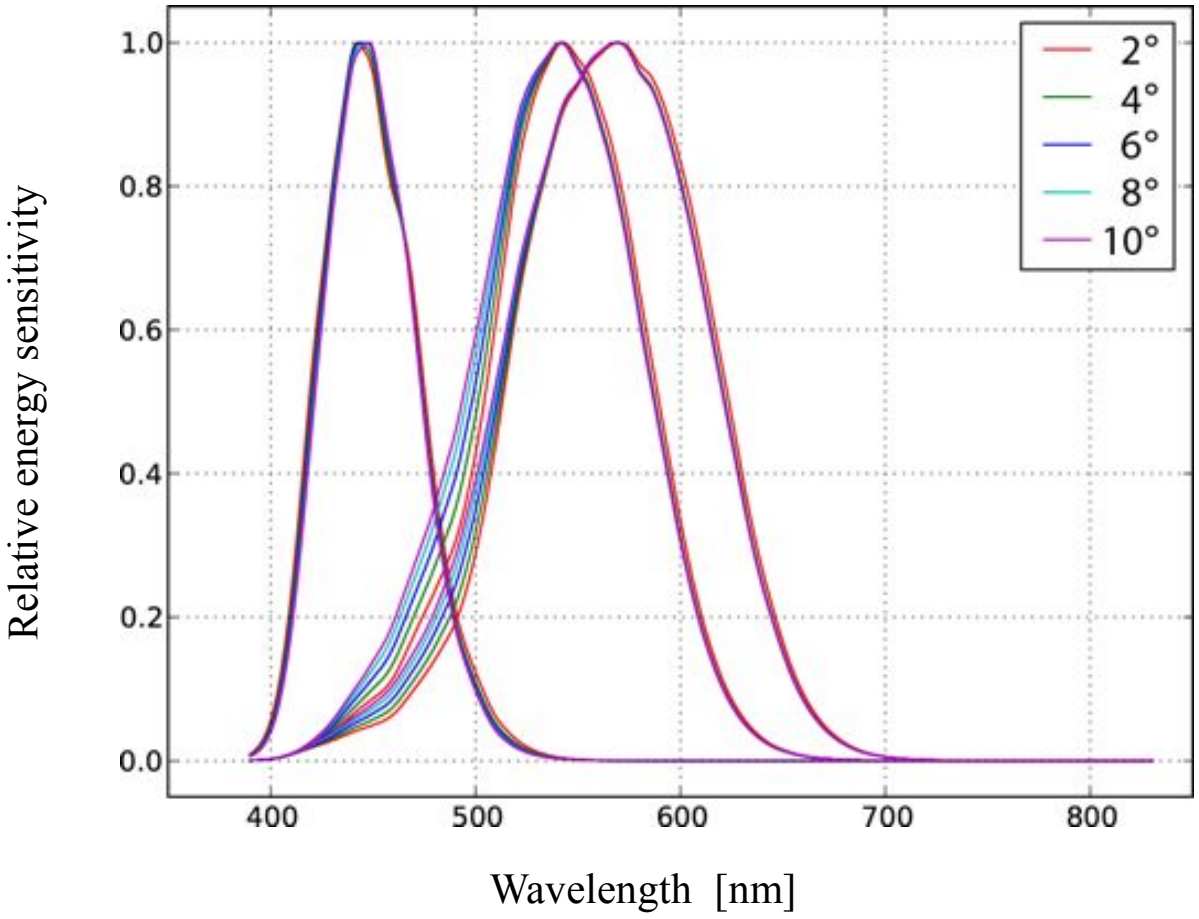
$$\begin{pmatrix} X_{\text{F, fs, age}} \\ Y_{\text{F, fs, age}} \\ Z_{\text{F, fs, age}} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{\text{fs, age}} \begin{pmatrix} L_{\text{fs, age}} \\ M_{\text{fs, age}} \\ S_{\text{fs, age}} \end{pmatrix}$$

↓ Colorimetrically based vector space

↓ Physiologically based vector space

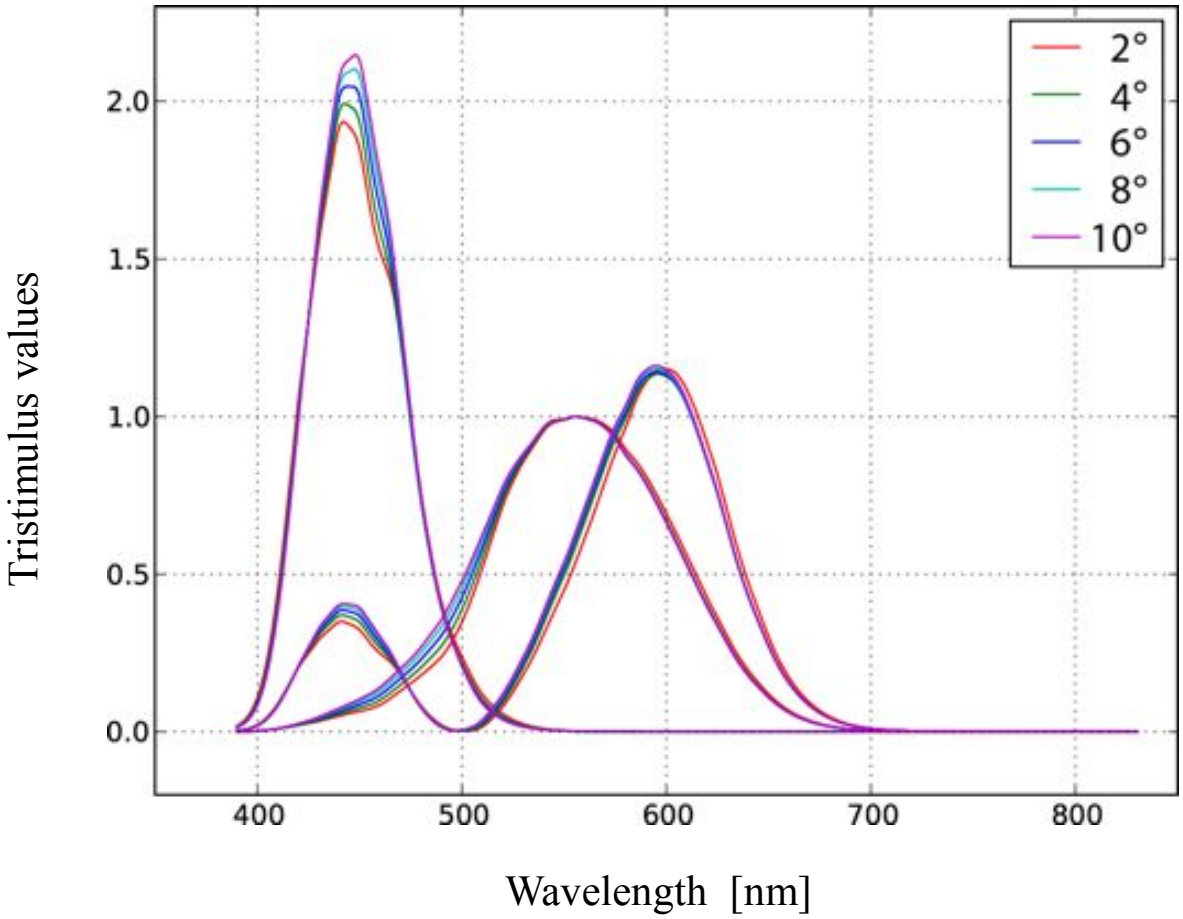
LMS cone fundamentals

(Different Field Sizes)



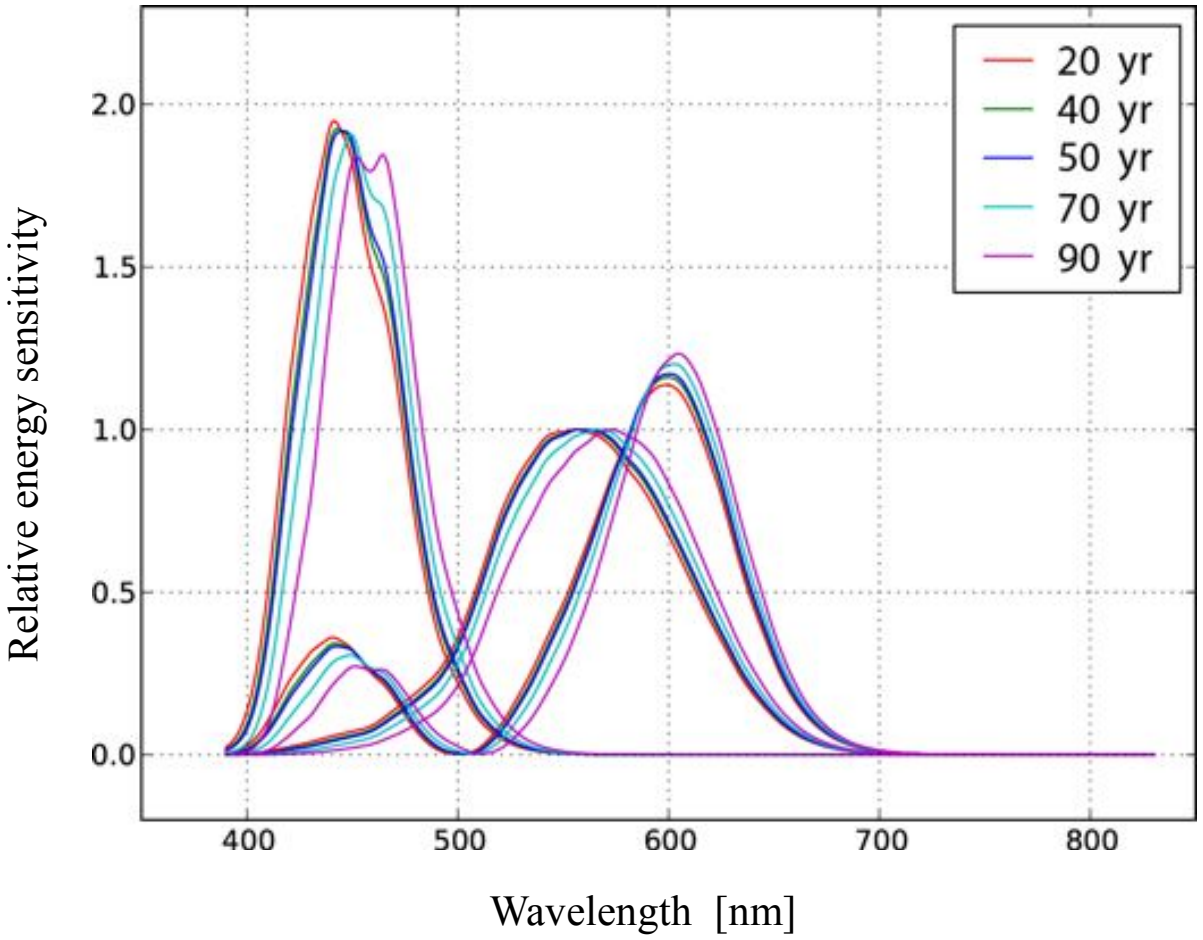
Fundamental XYZ colour-matching functions

(Different Field Sizes, Age 32)



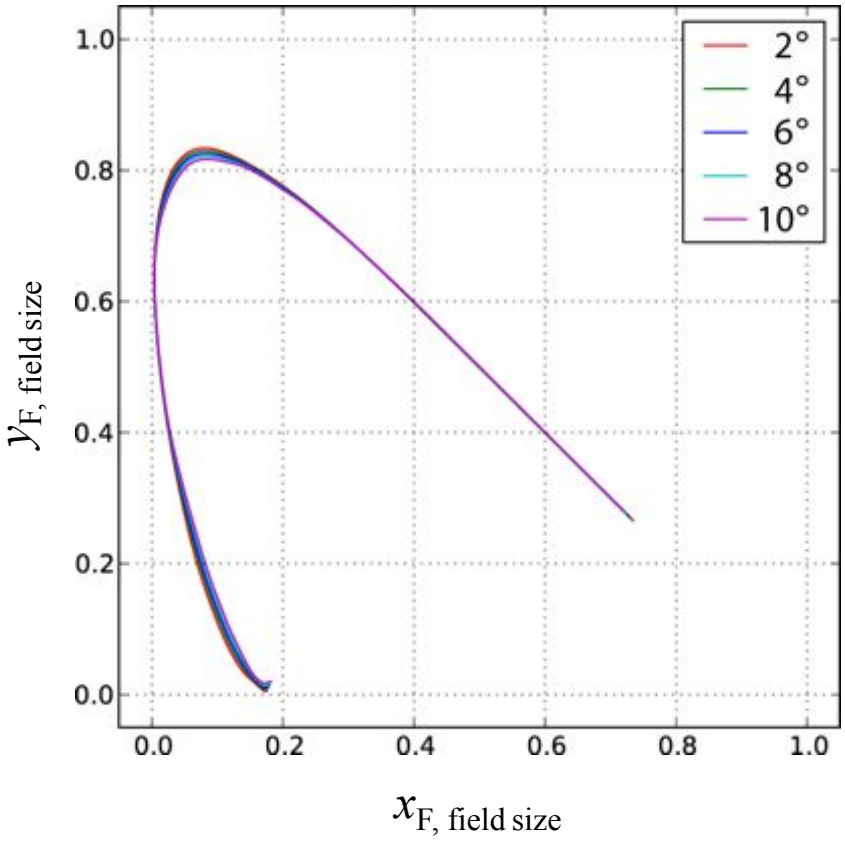
LMS cone fundamentals

(Different Ages)



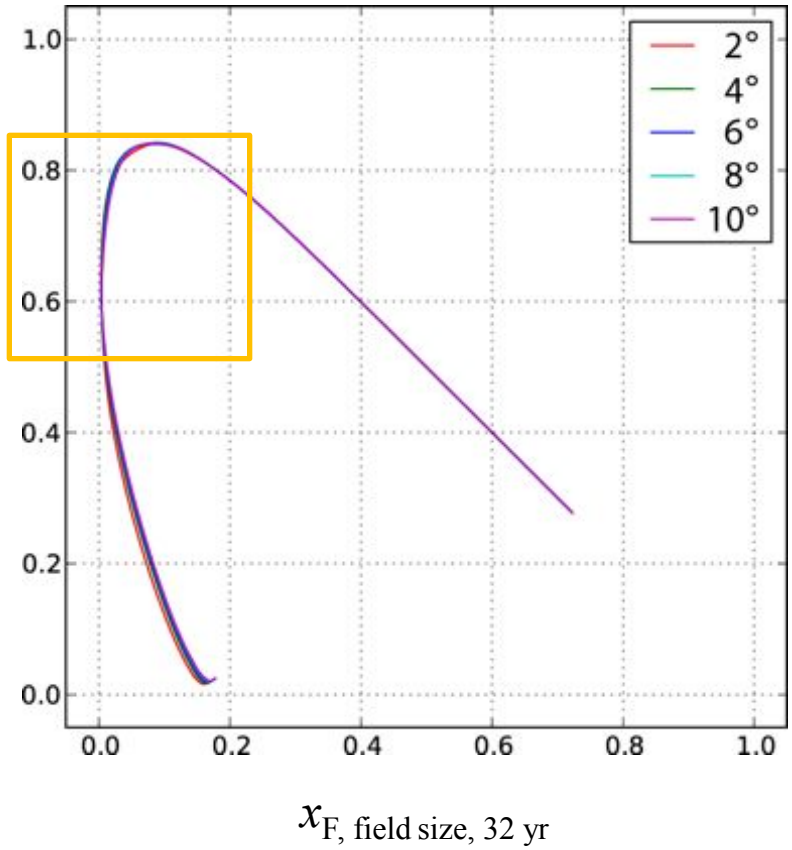
Reference Diagrams

(Morphed with Respect to Field Size)

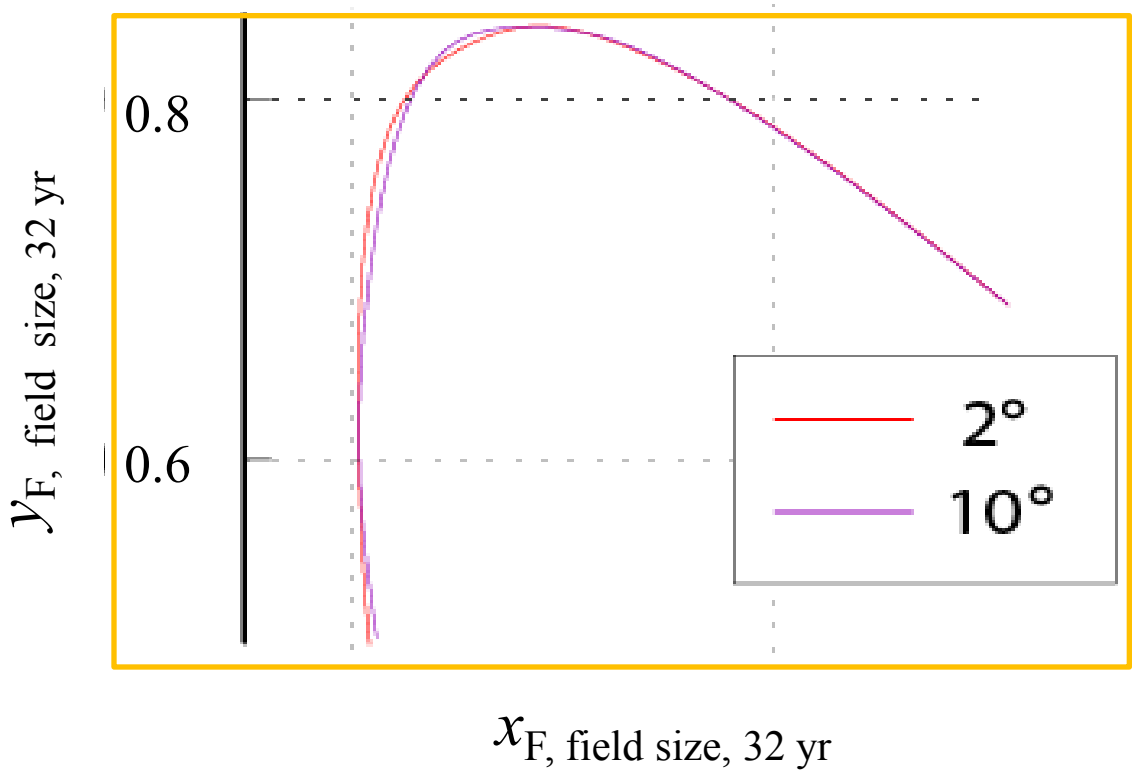


Optimized Fundamental xy Chromaticity Diagrams

(Different Field Sizes, Age 32)



The Spectrum Loci of the Fundamental *xy* Chromaticity Diagrams for 2º and 10º Field Size Intersect at Four Points (when plotted in a joint coordinate system)



Analyses show that, within the frame of the optimization concept, the intersections between the spectrum loci for different field sizes and ages is a characteristic of the cone fundamentals --not the transformations

Thank You