Towards A Unified Method For Computing Tristimulus Values

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Introduction

CIE in 1931 defined the TSV in terms of integrations. Here k is a scaling factor and for object colour it is defined by

$$X = k\int_a^b S(\lambda)\bar{x}(\lambda)R(\lambda)d\lambda$$

$$Y = k\int_a^b S(\lambda)\bar{y}(\lambda)R(\lambda)d\lambda$$

$$Z = k\int_a^b S(\lambda)\bar{z}(\lambda)R(\lambda)d\lambda$$

$$k = 100 / \int_a^b S(\lambda)\bar{y}(\lambda)d\lambda$$

(a,b) is the visible range of wavelength. From CIE documents a=360nm and b=830nm. For most applications where fluorescent is not involved, CIE [CIE pub 15.3] recommended a=380nm and b=780nm.

However, for most industrial applications, a=400nm and b=700nm.

Introduction

The problem with the TSV integrations is that there are no analytical expressions for all the integrands involved.

CIE defined some SPDs at 1nm intervals [CIE S014-2/E, CIE standard illuminants, 2007];

CIE defined CMFs at 1nm intervals [CIE S014-1/E, CIE standard observers, 2007];

R(λ): usually measured at interval > 1nm, such as 5nm, 10nm, or 20nm.

$$X = k\int_a^b S(\lambda)\bar{x}(\lambda)R(\lambda)d\lambda$$

$$Y = k\int_a^b S(\lambda)\bar{y}(\lambda)R(\lambda)d\lambda$$

$$Z = k\int_a^b S(\lambda)\bar{z}(\lambda)R(\lambda)d\lambda$$

Introduction

Hence, for computing the TSVs, numerical integrations must be used.

However, various numerical methods available, which method should be used?

Without a unified procedure, large disagreements in computing TSVs are possible, which cause problem.

This kind of error is completely avoidable if a unified method is used, even it is not a good one.

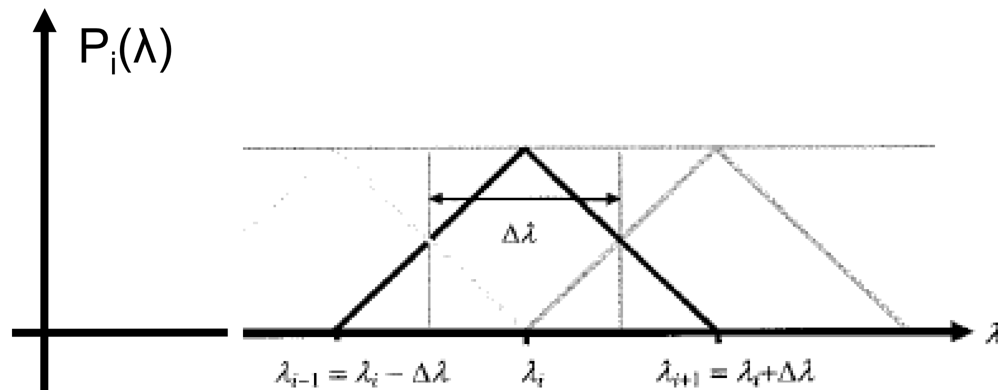
Stearns and Stearns Correction [CRA,1988]:

It was assumed that Suppose $R(\lambda) = c_0 + c_1(\lambda - \lambda_i) + c_2(\lambda - \lambda_i)^2$ over $[\lambda_{i-1}, \lambda_{i+1}]$, and

the measured reflectance $\hat{r}_i^{(\Delta\lambda)}$ at λ_i satisfies:

$$\hat{r}_i^{(\Delta\lambda)} = \int_a^b P_i(\lambda) R(\lambda) d\lambda$$

where



Measurement Corrections

Thee Terms Stearns & Stearns Correction [CRA, 1988]

$$
\begin{aligned}
R(\lambda_0) &= (1+\alpha)\,\hat{r}_0^{(\Delta\lambda)} - \alpha\,\hat{r}_1^{(\Delta\lambda)} \\
R(\lambda_i) &= -\alpha\,\hat{r}_{i-1}^{(\Delta\lambda)} + (1+2\alpha)\,\hat{r}_i^{(\Delta\lambda)} - \alpha\,\hat{r}_{i+1}^{(\Delta\lambda)} \\
R(\lambda_n) &= -\alpha\,\hat{r}_{n-1}^{(\Delta\lambda)} + (1+\alpha)\,\hat{r}_n^{(\Delta\lambda)}
\end{aligned}
$$

Here α =1/10, or 1/12, or 0.083 .

Fairman Correction [CRA, 2010]

1. For the first and last measured bandpasses, no correction.

$$R_{s,0} = R_{m,0}$$
$$R_{s,n} = R_{m,n}$$

2. For the second and next-to-last passbands, a four-point correction is to be made.

$$R_{s,0} = -0.10R_{m,-1} + 1.21R_{m,0} - 0.12R_{m,1} + 0.01R_{m,2}.$$

3. For all remaining interior passbands, the following five-point correction is to be made.

$$R_{s,0} = 0.01R_{m,-2} - 0.12R_{m,-1} + 1.22R_{m,0} - 0.12R_{m,1} + 0.01R_{m,2}.$$

Interpolations

CIE pub 15.2, 2004 :

Third order Largrange interpolation

CIE pub 167, 2005 :

① 5th order Sprague interpolation for uniform sampling data

② Third order Spline interpolation for non-uniform sampling data

CIE Recommendations

[pub 15, 1971; pub 15.2, 1986; pub15.3, 2004]:

- 1nm summations:

  1 The standard method for evaluating these integrals is numerical summation from 360 nm to 830 nm at wavelength intervals, Δλ equal to 1 nm according to the equations:

$$X = k\sum_{i=0}^{n} S(\lambda_i)\bar{x}(\lambda_i)R(\lambda_i)\Delta\lambda$$

$$Y = k\sum_{i=0}^{n} S(\lambda_i)\bar{y}(\lambda_i)R(\lambda_i)\Delta\lambda$$

$$Z = k\sum_{i=0}^{n} S(\lambda_i)\bar{z}(\lambda_i)R(\lambda_i)\Delta\lambda$$

$$\Delta\lambda = 1$$

$$k = 100 / \sum_{i=0}^{n} S(\lambda_i)\bar{y}(\lambda_i)\Delta\lambda$$

  2 The fundamental colorimetric tables are 1nm tables in CIE standards. All rigorous calculations should use these 1nm tables for most practical purposes, the summation may be approximated by using wavelength intervals Δλ equal to 5nm over the wavelength range 380nm to 780nm,….

CIE Recommendations

[pub15.3, 2004]:

- For larger than 5nm intervals:
  1. Data can be interpolated into 1nm interval and then use the 1nm summation formula.

     For this option it is named as **CIE-R method**
  2. ASTM weighting tables can be used for 10 & 20nm data;
  3. Li, Luo and Rigg method [2004, CR&A] can be used.

Direct Selection Method

$$X = k \sum_{\lambda=a \;\; step \;\; \Delta\lambda}^{b} S(\lambda)\bar{x}(\lambda)R(\lambda)\Delta\lambda$$

$$Y = k \sum_{\lambda=a \;\; step \;\; \Delta\lambda}^{b} S(\lambda)\bar{y}(\lambda)R(\lambda)\Delta\lambda$$

$$Y = k \sum_{\lambda=a \;\; step \;\; \Delta\lambda}^{b} S(\lambda)\bar{z}(\lambda)R(\lambda)\Delta\lambda$$

For example, Δλ=10nm

$$k = 100/\{10*[S(380)\bar{y}(380) + S(390)\bar{y}(390) + \cdots + S(780)\bar{y}(780)]\}$$

$$X = 10k[S(380)\bar{x}(380)R(380) + S(390)\bar{x}(390)R(390) + \cdots + S(780)\bar{x}(780)R(780)]$$
$$Y = 10k[S(380)\bar{y}(380)R(380) + S(390)\bar{y}(390)R(390) + \cdots + S(780)\bar{y}(780)R(780)]$$
$$Z = 10k[S(380)\bar{z}(380)R(380) + S(390)\bar{z}(390)R(390) + \cdots + S(780)\bar{z}(780)R(780)]$$

ASTM E-308-1985 Weighting Tables

For a unified way for computing TSVs, ASTM standardised a set of weighting tables in 1985:

$$W_{X,j}^{(\Delta\lambda)},\ W_{Y,j}^{(\Delta\lambda)},\ W_{Z,j}^{(\Delta\lambda)},\ j = 0,1,\cdots,n$$

under 9 illuminations and two standard observers between 360nm and 780nm at 10nm and 20nm intervals. Altogether 36 weighting tables were defined, which is known now as **Table 5**.

For a given/measured and bandpass corrected reflectance $\hat{r}_j^{(\Delta\lambda)}$, j=1,2,..,n, the TSV can be computed using:

$$X = \sum_{j=0}^{n} W_{X,j}^{(\Delta\lambda)} \hat{r}_j^{(\Delta\lambda)}, \qquad Y = \sum_{j=0}^{n} W_{Y,j}^{(\Delta\lambda)} \hat{r}_j^{(\Delta\lambda)}, \qquad Z = \sum_{j=0}^{n} W_{Z,j}^{(\Delta\lambda)} \hat{r}_j^{(\Delta\lambda)}$$

ASTM E-308-1995 Weighting Tables

In 1995, an updated set of weighting tables was defined based on **Venable** method [CRA, 1989] and Stearns and Stearns corrections.

This set is known now as Table 6.

However, this set of weighting tables must be used directly with the measured reflectance since the bandpass correction was built in the derivation of weighting tables.

Oleari Method

Oleari [CR&A, 2000] developed a new method for computing TSVs from measured reflectance functions and reported that the new method is better than ASTM weighting tables.

Local Power Expansions for the Products of the SPD and CMF

1) compute: $x_{i,0}, x_{i,1}, x_{i,2}$ for $i = 0,1,\ldots,n$

Assume $S(\lambda)\bar{x}(\lambda) = x_{i,0} + x_{i,1}(\lambda - \lambda_i) + x_{i,2}(\lambda - \lambda_i)^2$ for

$\lambda \in [\lambda_i - \Delta\lambda, \lambda_i + \Delta\lambda]$

The coefficients can be found from minimising:

$$\varphi(x_{i,0}, x_{i,1}, x_{i,2}) = \sum_{j=-\Delta\lambda}^{\Delta\lambda} [S(\lambda_i + j)\bar{x}(\lambda_i + j) - (x_{i,0} + x_{i,1} j + x_{i,2} j^2)]^2$$

Oleari Method

$$x_{i,0} = \frac{1}{A}[B_0(\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^4) - B_2(\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^2)\}$$

$$x_{i,1} = B_1 \Big/ \{\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^2\}$$

$$x_{i,2} = \frac{1}{A}[-B_0(\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^2) + B_2(2\Delta\lambda + 1)]$$

with

$$A = (2\Delta\lambda + 1)\{\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^4\} - [\sum_{j=-\Delta\lambda}^{\Delta\lambda} j^2]^2$$

$$B_0 = \sum_{j=-\Delta\lambda}^{\Delta\lambda} [S(\lambda_i + j)\bar{x}(\lambda_i + j)]$$

$$B_1 = \sum_{j=-\Delta\lambda}^{\Delta\lambda} \{[S(\lambda_i + j)\bar{x}(\lambda_i + j)]j\}$$

$$B_2 = \sum_{j=-\Delta\lambda}^{\Delta\lambda} \{[S(\lambda_i + j)\bar{x}(\lambda_i + j)]j^2\}$$

Similarly, $y_{i,0}$, $y_{i,1}$, $y_{i,2}$ for $i = 0,1,\ldots,n$ can be computed from local power expansion of $S(\lambda)\bar{y}(\lambda)$; and $z_{i,0}$, $z_{i,1}$, $z_{i,2}$ for $i = 0,1,\ldots,n$, can be computed from local power expansion of .

2） Compute X, Y, Z,

Set $$H=\begin{pmatrix} -1/12 & 14/12 & -1/12 \\ -1/(2\Delta\lambda) & 0 & 1/(2\Delta\lambda) \\ 1/[2(\Delta\lambda)^2] & -1/(\Delta\lambda)^2 & 1/[2(\Delta\lambda)^2] \end{pmatrix},$$

$$k=100/\{\Delta\lambda\sum_{i=0}^{N}\begin{pmatrix} 1 & \frac{(\Delta\lambda)^2}{6} \end{pmatrix}\begin{pmatrix} y_{i,0} & 0 & 0 \\ y_{i,2} & y_{i,1} & y_{i,0} \end{pmatrix} H \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\}$$

$$X=k\Delta\lambda\sum_{i=0}^{N}\begin{pmatrix} 1 & \frac{(\Delta\lambda)^2}{6} \end{pmatrix}\begin{pmatrix} x_{i,0} & 0 & 0 \\ x_{i,2} & x_{i,1} & x_{i,0} \end{pmatrix} H \begin{pmatrix} \hat{r}_{i-1}^{(\Delta\lambda)} \\ \hat{r}_{i}^{(\Delta\lambda)} \\ \hat{r}_{i+1}^{(\Delta\lambda)} \end{pmatrix}$$

$$Y=k\Delta\lambda\sum_{i=0}^{N}\begin{pmatrix} 1 & \frac{(\Delta\lambda)^2}{6} \end{pmatrix}\begin{pmatrix} y_{i,0} & 0 & 0 \\ y_{i,2} & y_{i,1} & y_{i,0} \end{pmatrix} H \begin{pmatrix} \hat{r}_{i-1}^{(\Delta\lambda)} \\ \hat{r}_{i}^{(\Delta\lambda)} \\ \hat{r}_{i+1}^{(\Delta\lambda)} \end{pmatrix}$$

$$Z=k\Delta\lambda\sum_{i=0}^{N}\begin{pmatrix} 1 & \frac{(\Delta\lambda)^2}{6} \end{pmatrix}\begin{pmatrix} z_{i,0} & 0 & 0 \\ z_{i,2} & z_{i,1} & z_{i,0} \end{pmatrix} H \begin{pmatrix} \hat{r}_{i-1}^{(\Delta\lambda)} \\ \hat{r}_{i}^{(\Delta\lambda)} \\ \hat{r}_{i+1}^{(\Delta\lambda)} \end{pmatrix}$$

Oleari Method

In 2011, Li et al (JOSA) re-formulated the Oleari method and gave two sets of weighting tables. One is named as **zero-order** (OWT0) and the other the second order (OWT2).

Similar to the ASTM weighting tables, the Zero order tables must be used with reflectance corrected using SS formulae and the Second order weighting tables must be used with the measured reflectance directly.

Li, Luo and Rigg (LLR) Method

[CRA,2004] Computing weighting tables

$$W_{X,j}^{(\Delta\lambda)},\ W_{Y,j}^{(\Delta\lambda)},\ W_{Z,j}^{(\Delta\lambda)},\ j=0,1,\cdots,n$$

by solving three linear system of equations:

$$Au_V^{(\Delta\lambda)} = a_V^{(\Delta\lambda)}$$

$$A=\begin{pmatrix} 5 & 1 & & & & \\ 1 & 4 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & 1 & 4 & 1 \\ & & & & 1 & 5 \end{pmatrix},\quad u_V^{(\Delta\lambda)}=\begin{pmatrix} W_{V,0}^{(\Delta\lambda)} \\ W_{V,1}^{(\Delta\lambda)} \\ \vdots \\ W_{V,n}^{(\Delta\lambda)} \end{pmatrix},\quad a_V^{(\Delta\lambda)}=\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix},$$

$$a_j = 6\Delta\lambda \int^{\lambda_{j+1}} W_V(\lambda)P_j(\lambda)d\lambda, \quad \text{where} \qquad W_V(\lambda) = kS(\lambda)\bar{v}(\lambda)$$

Li, Wang and Luo (LWL) Method

[SPIE,2006] Computing weighting tables

$$W_{X,j}^{(\Delta\lambda)},\ W_{Y,j}^{(\Delta\lambda)},\ W_{Z,j}^{(\Delta\lambda)},\ j=0,1,\cdots,n$$

by solving three linear system of equations:

$$Bu_V^{(\Delta\lambda)} = a_V^{(\Delta\lambda)}$$

$$B = \begin{pmatrix} c_1 & c_3 & & & \\ c_3 & c_2 & c_3 & & \\ & \ddots & \ddots & \ddots & \\ & & c_3 & c_2 & c_3 \\ & & & c_3 & c_1 \end{pmatrix},\quad u_V^{(\Delta\lambda)} = \begin{pmatrix} W_{V,0}^{(\Delta\lambda)} \\ W_{V,1}^{(\Delta\lambda)} \\ \vdots \\ W_{V,n}^{(\Delta\lambda)} \end{pmatrix}$$

with $c_1 = \dfrac{5(\Delta\lambda)^2+1}{(\Delta\lambda)^2}$, $c_2 = \dfrac{4(\Delta\lambda)^2+2}{(\Delta\lambda)^2}$, $c_3 = \dfrac{(\Delta\lambda)^2-1}{(\Delta\lambda)^2}$

Comparisons

Up to now, we have reviewed available methods for computing the TSVs. Now we are ready to compare them using the simulation method used by Fairman [CR&A, 1995].

Comparison Procedures

1) 1nm Standard reflectance and Ground Truth TSVs

a) The 1096 1nm reflectance values measured from Pantone colour chips provided by Danny Rich are used as standard reflectances.

b) Wavelength range for the comparison is from 360nm to 780nm.

c) Using 1nm SPD and CMF, the TSVs can be computed, which are called the **Ground Truth** TSVs.

Comparisons

2) Simulated measured reflectance functions at 10nm and 20nm

For each of the 1nm standard reflectance R(λ), the measured reflectance values can be computed using:

$$\hat{r}_i^{(\Delta\lambda)} = \int_a^b P_i(\lambda) R(\lambda) d\lambda$$

which can be computed using 1nm summation. Thus, 1096 10nm and 20nm measured reflectance functions were obtained.

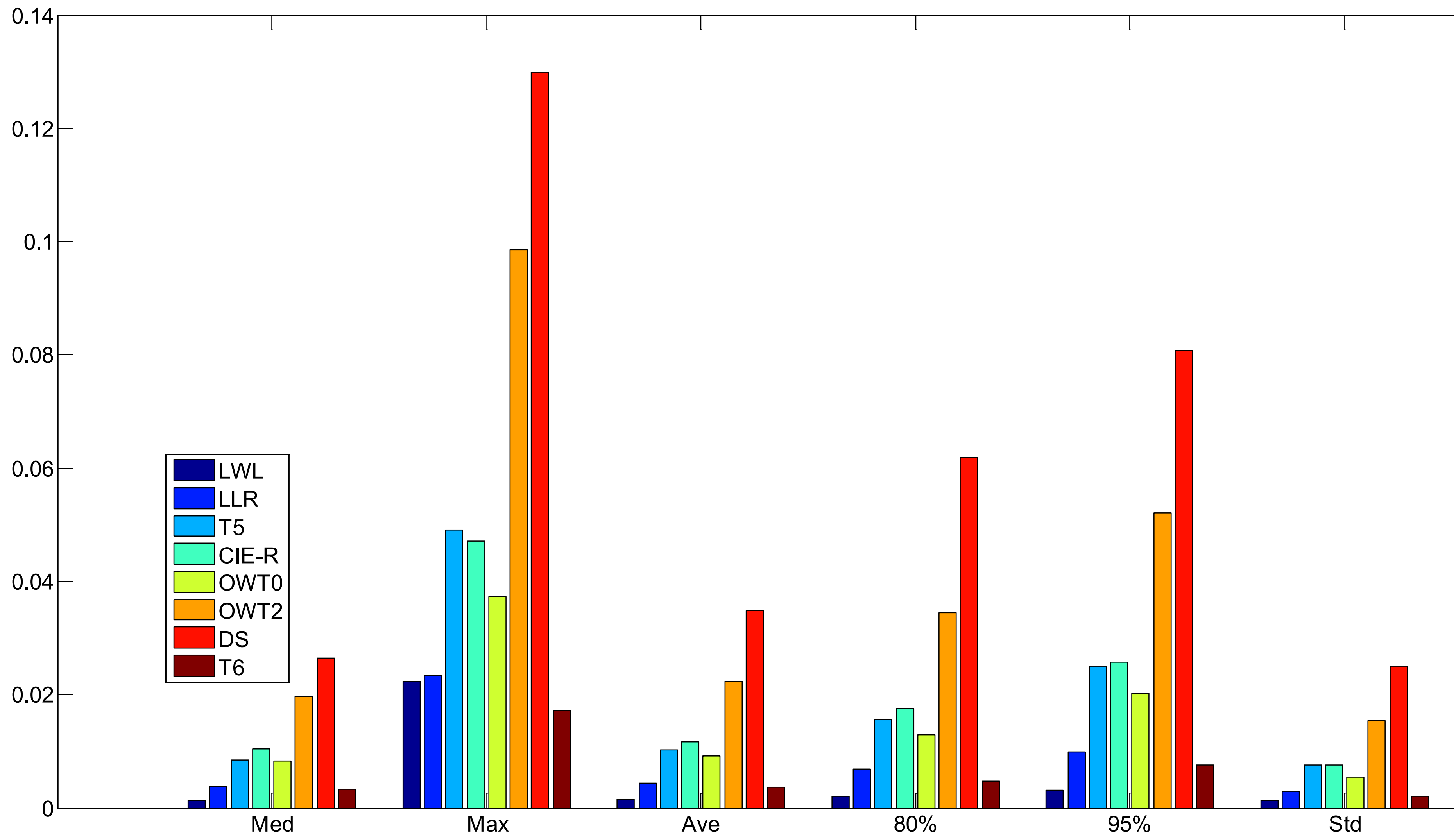
3) Compute TSVs using the measured 10nm and 20nm reflectance

TSVs can be computed using each of methods/weighting tables and the measured 10 or 20 nm reflectance functions obtained in Step 2).

4) Compute Colour Differences

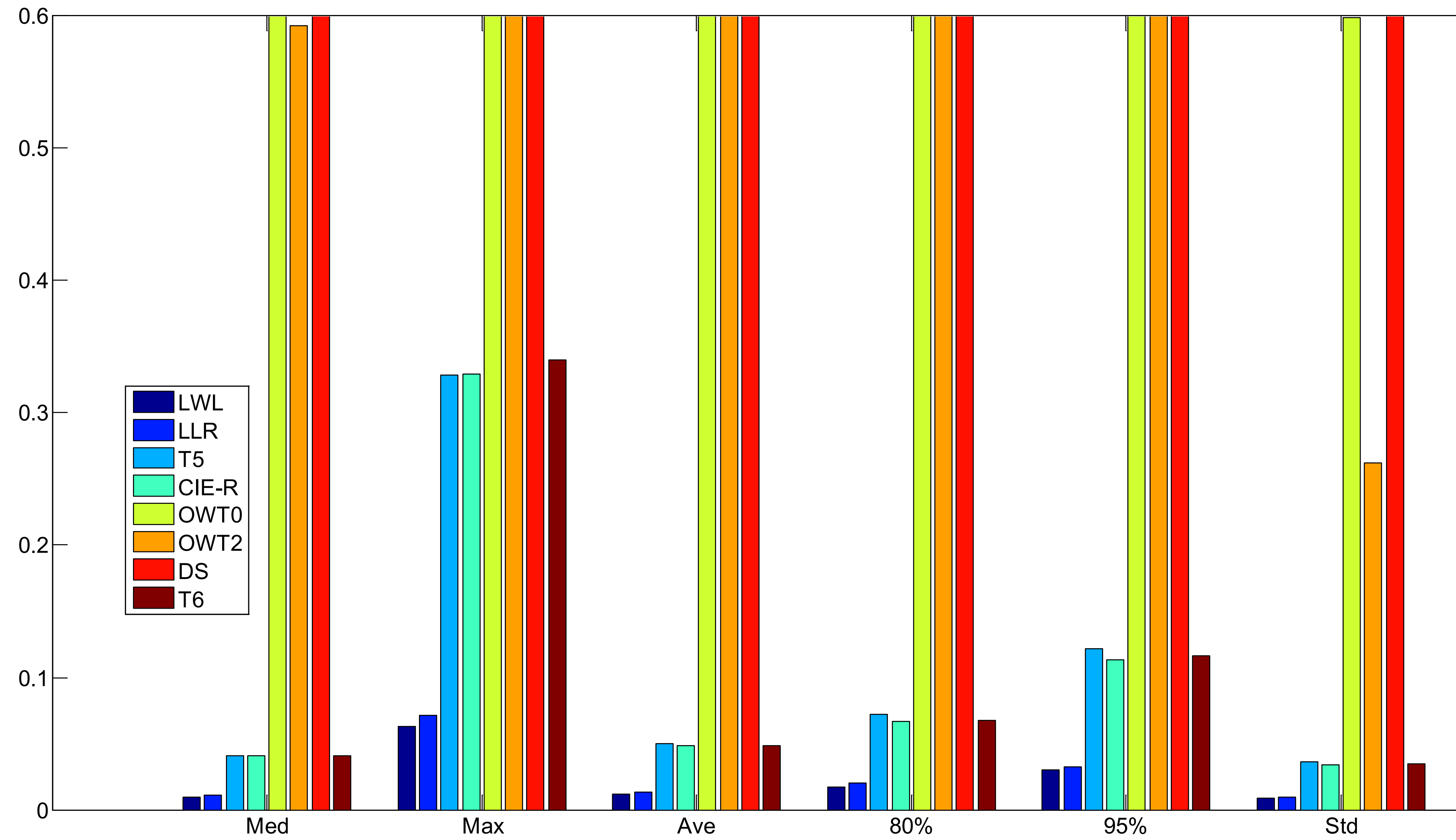
The colour differences between the TSVs computed in Step 3) and GT TSVs computed in Step 1) can be computed. The smaller the colour difference is, the better the weighting table/Method used. Since there are more colour differences computed, median / Maximum and other statistical measures were reported.

Results



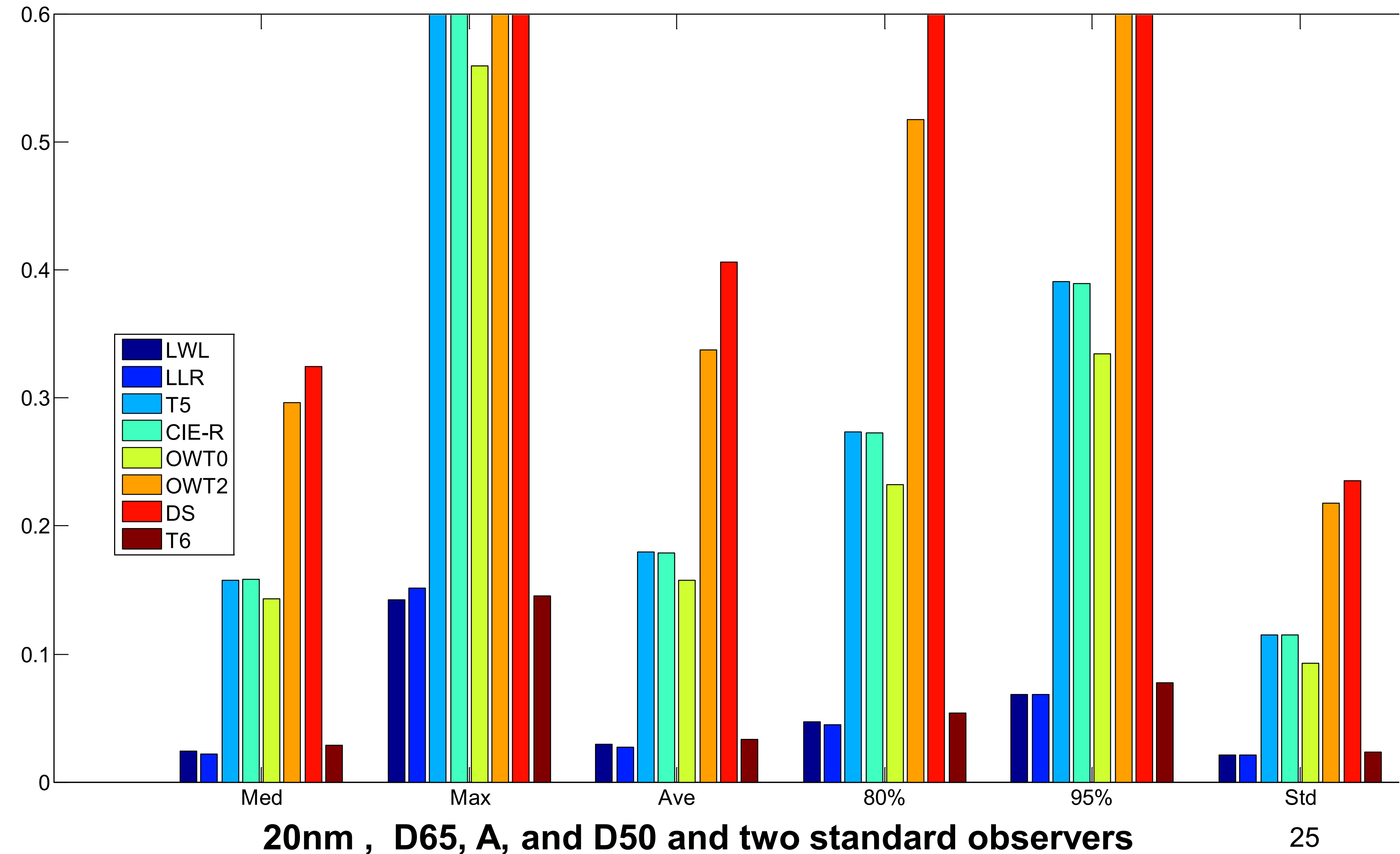
10nm， D65, A, and D50 and two standard observers

Results



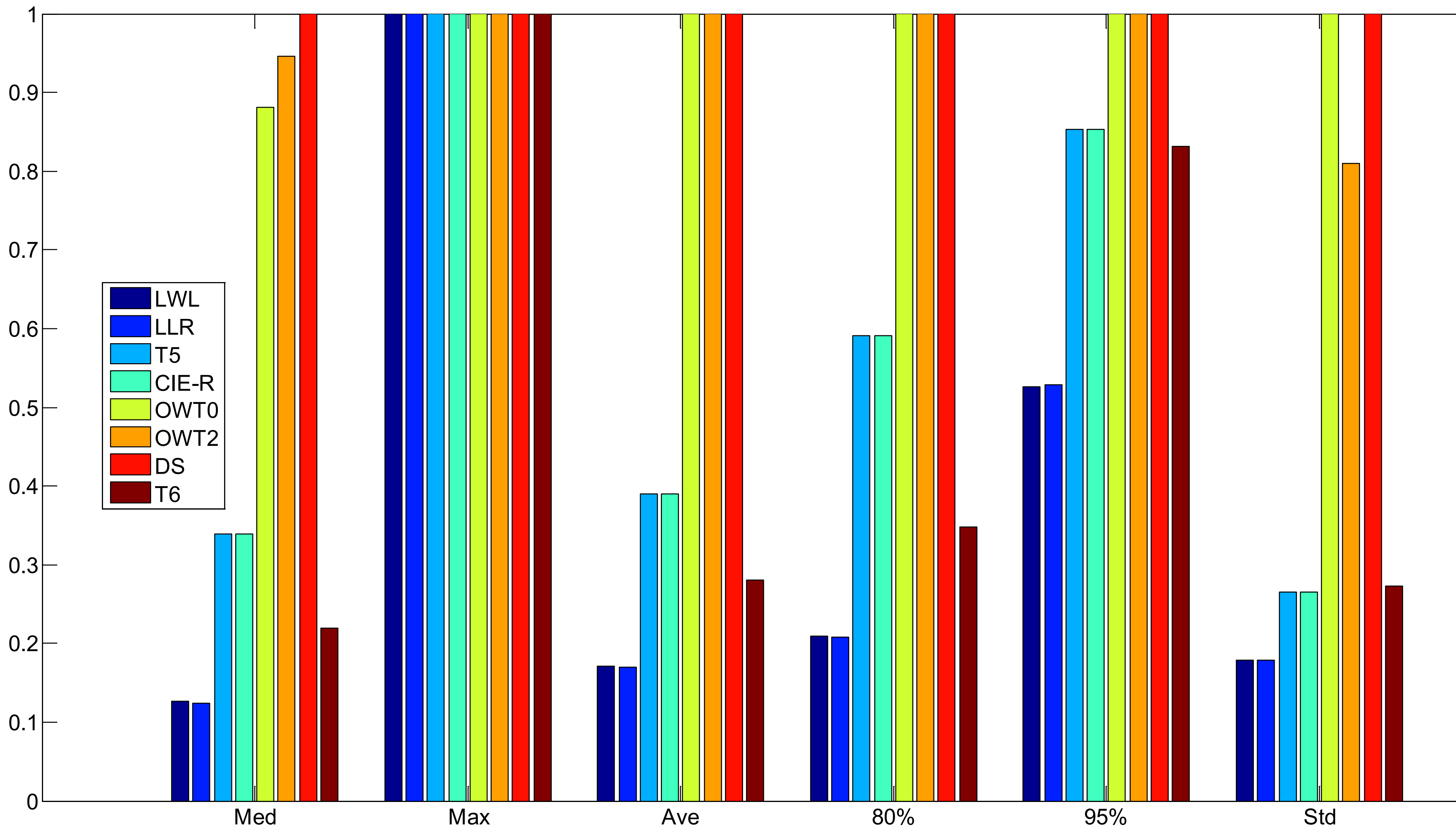
10nm， F2, F7, and F11 and two standard observers

Results



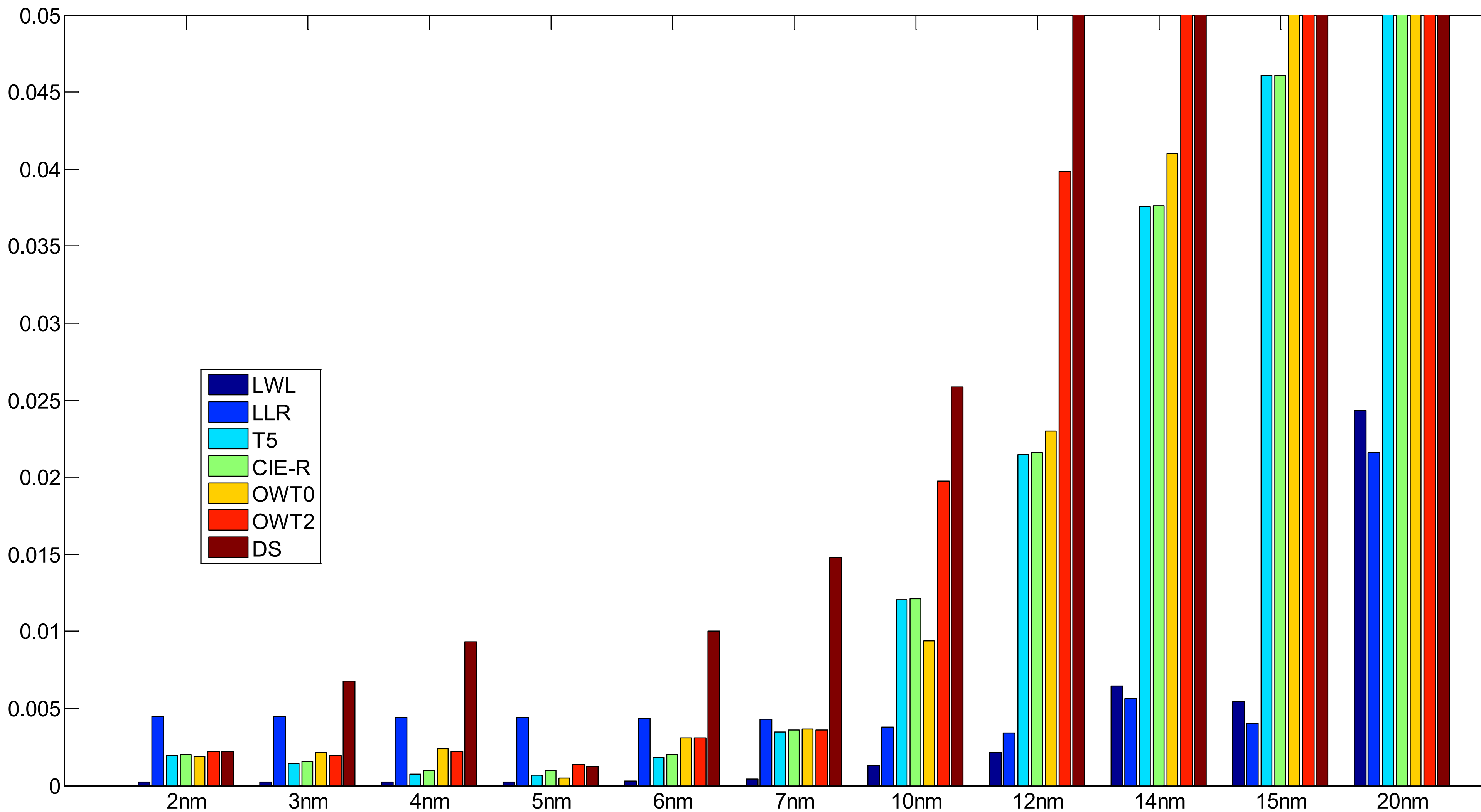
20nm， D65, A, and D50 and two standard observers

Results



20nm， F2, F7, and F11 and two standard observers

Results



Median CD for Continuous Illuminants & two observers

Results

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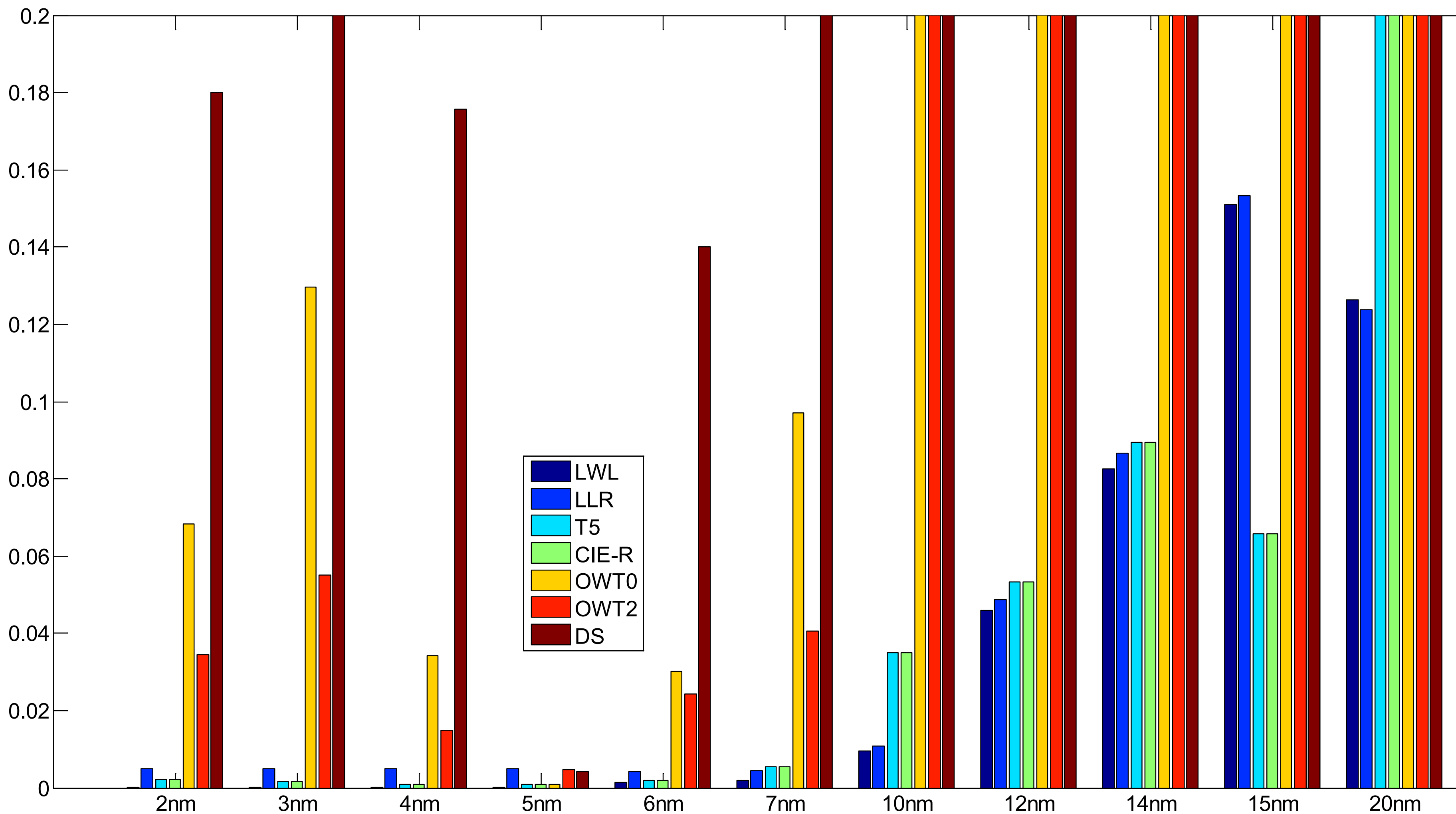

Median CD for Fluorescent Illuminants & two observers

Conclusions

According to the tests:

1) LWL is the best for intervals less than 12nm
2) LLR method is the best for interval lengths greater than 12nm and the LWL is the second best

It is therefore, the LWL method is proposed for computing TSVs for any measuring interval length greater than 1nm for three reasons:

- ✓ The method is more accurate;
- ✓ The method is simple to implement;
- ✓ "There should be no reason for the introduction of significant errors in this calculation process, even though other errors, such as those of specimen preparation or measurement, may be large in comparison." [Billmeyer& Fairman CRA, 1987]