

# The Slope of Self-luminous Neutral Scale: Independent Converging Evidence

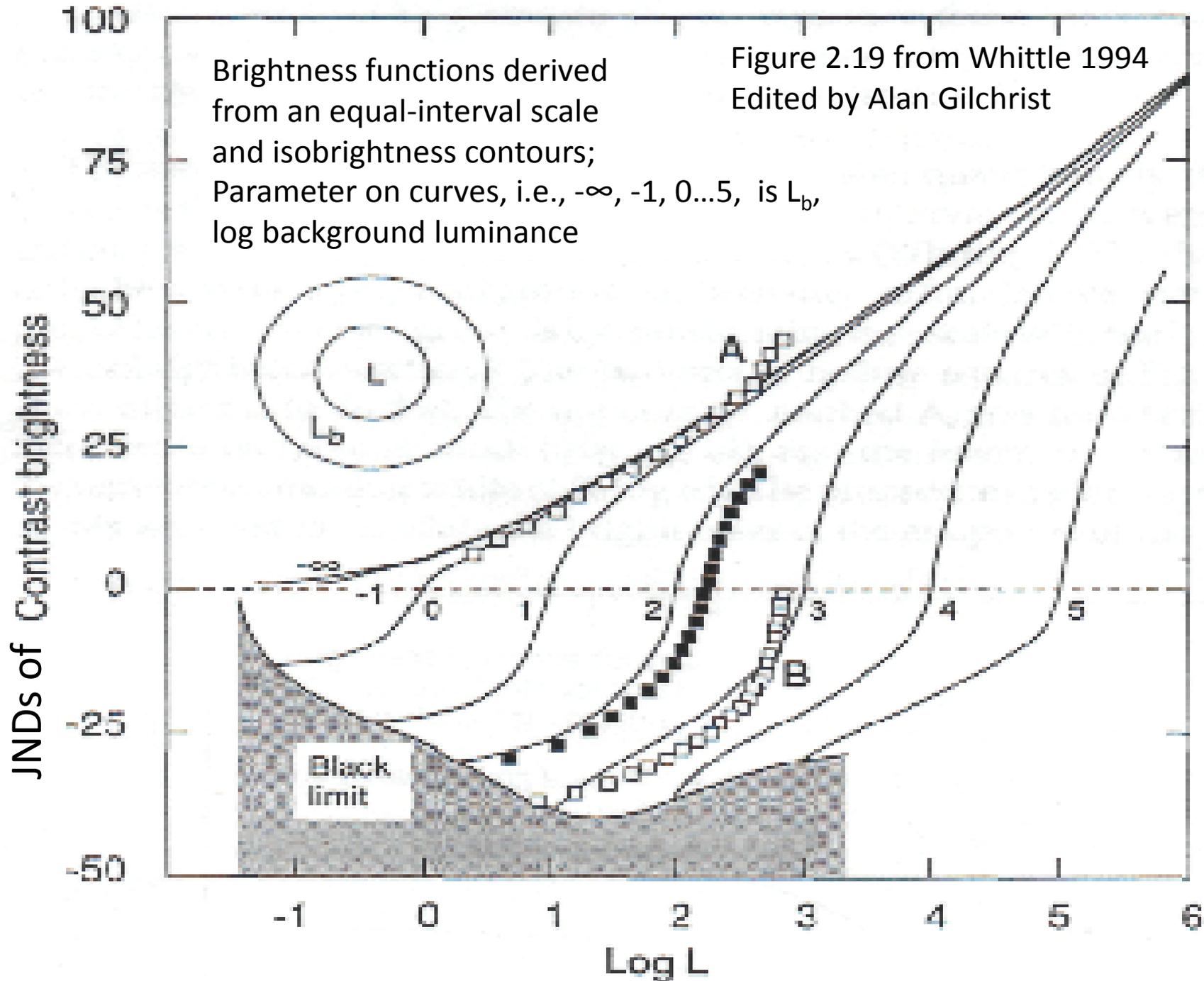
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With acknowledgement of  
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Figure 2.19 from Whittle 1994  
Edited by Alan Gilchrist

Brightness functions derived from an equal-interval scale and isobrightness contours; Parameter on curves, i.e.,  $-\infty, -1, 0 \dots 5$ , is  $L_b$ , log background luminance



# Noticeable Changes of Brightness in context

- Paul Whittle 1992: Settings of color-neutral differences “just noticeable at a glance” (JND)
- A critical context is minimum {background or negative contrast} luminance (L).

JND = F[W= $\Delta L / (L_{\text{background}})$ ] for + contrast

JND = G[W= $\Delta L / (L_{\text{target}})$ ] for negative contrast

$$W = \Delta L / L_{\text{min}}$$



“...the simplest and most precise  
mathematical description...”

$L$  is target luminance,  $L_b$  is background luminance

$L_{\min}$  = the minimum of  $L$  or  $L_b$

$$\Delta L = |L - L_b|$$

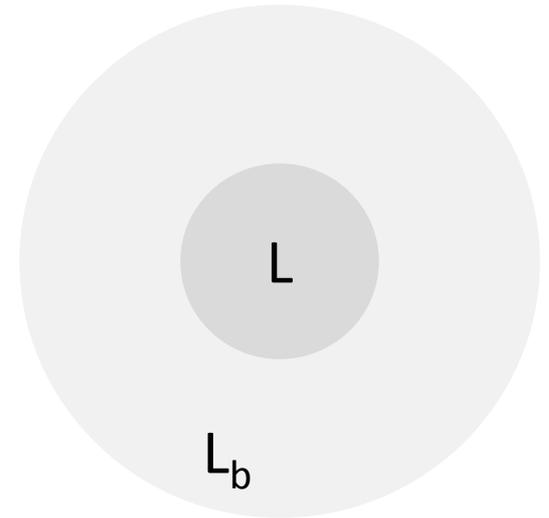
The number of JNDs between  $L_b$  and  $L$   
=  **$a \log(1 + bW)$** ,

Two cases (e.g., negative contrast case is  
shown in figure):

For negative contrast:  $W = (1-k) \Delta L / (L_d + L_{\min} + k \Delta L)$ ;  $a = -7.07$

For positive contrast:  $W = (1-k) \Delta L / (L_d + L_{\min})$ ;  $a = 8.22$

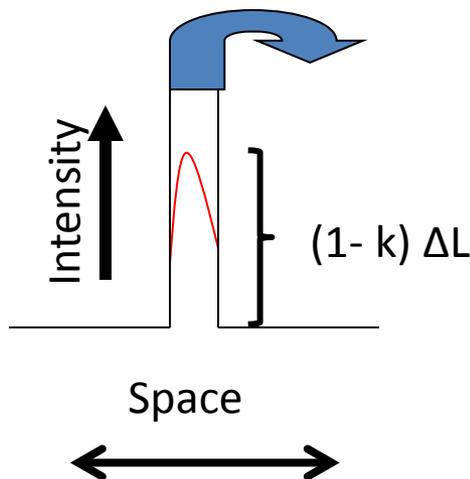
where  $L_d = .39 \text{ cd/m}^2$  represents the black limit,  $b = 6.58$ , and  
 $k$  ( $0 < k < 1$ ) represents intraocular light scattering and grows with  
decreasing subtense.



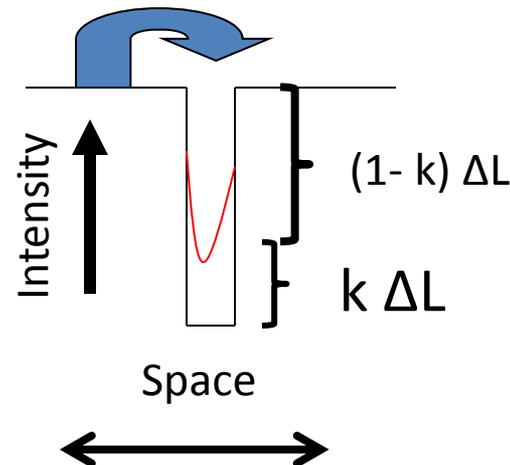
# Explaining $0 < k < 1$ in the Whittle formula: Before the retina, **Intraocular Scattering** **Reduces Contrast**

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- **Light** from a positive contrast (a target more-intense than its surround) is scattered out of the target by the intraocular media:



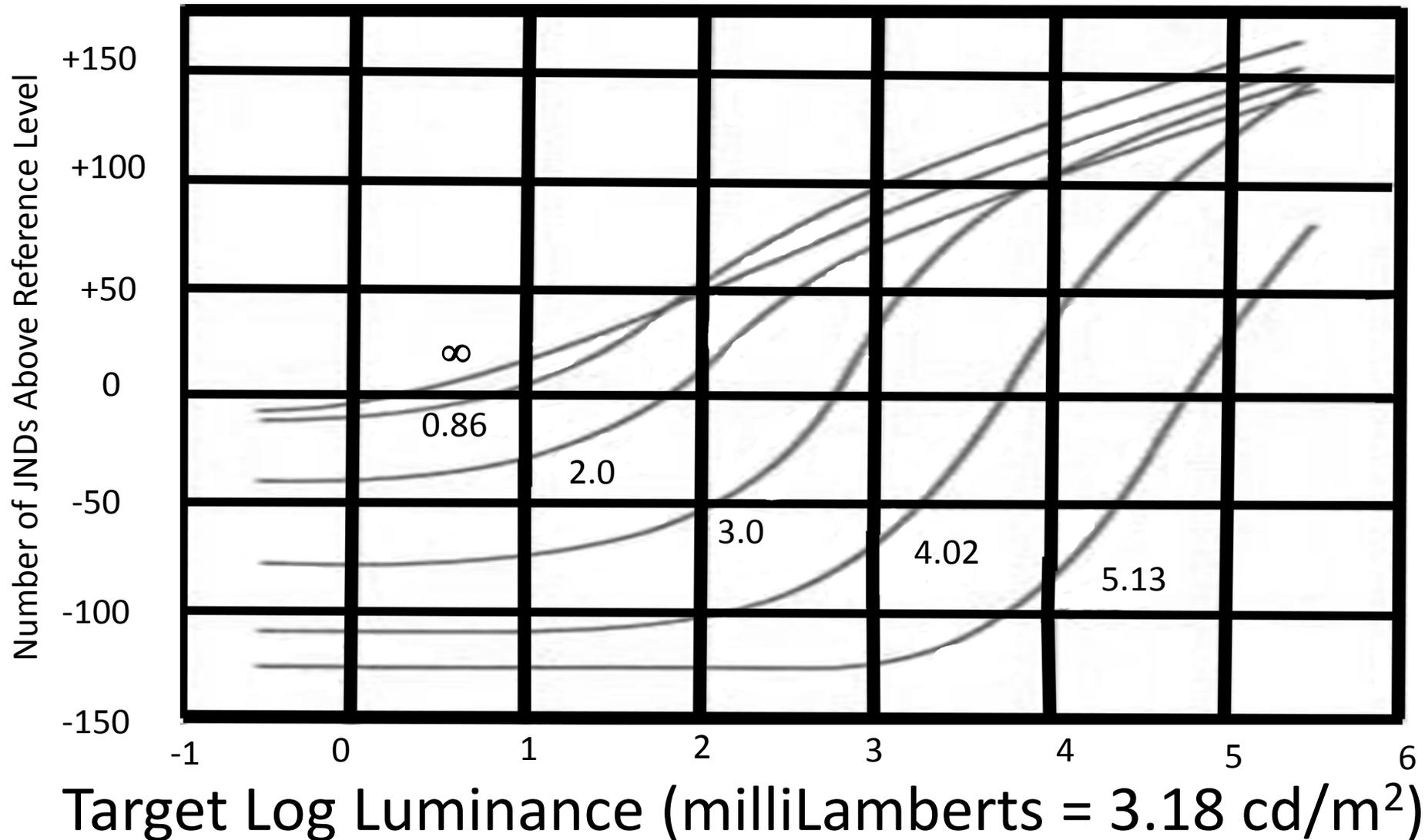
- **Light** from the surround of a negative contrast (a target less-intense than its surround) is scattered into the target by the intraocular media:



# Slope of Whittle's Formula, JND

- $JND = a \log(1+bW)$
- $d(JND)/dL = JND' =$   
limit (as  $\Delta L$  approaches 0)  $\{JND(L + \Delta L) - JND(L)\} / \Delta L$
- Approximated with  $\Delta L$  a constant on the order of threshold  $\Delta L$ .
- We will plot these slopes against independently estimated slopes of the neutral scale...

# Heinemann's (1972) cumulative gray-scale JND curves (Parameter on curves is log background luminance.)



# Heinemann's Slope: $dJND/d(\log Y)$

Heinemann's 1972 Claim:  $dJND/d(\log L) = L/\Delta L$ , where  $\Delta L$  is increment threshold.

Our Derivation: Start at left, apply **chain rule** & recall  $d \log(L)/dL = 1/L$ .

$$\begin{aligned}dJND/d(\log L) &= [dJND/dL][dL/d(\log(L))] \\ &= [dJND/dL] / [d(\log(L))/dL] \\ &= [dJND/dL]L\end{aligned}$$

approximate  $dJND$  &  $dL$  by  $\Delta JND$  &  $\Delta L$ .

Set  $\Delta JND$  as the **unit** of the gray scale...

$$dJND/dL \sim \Delta JND/\Delta L = 1/\Delta L.$$

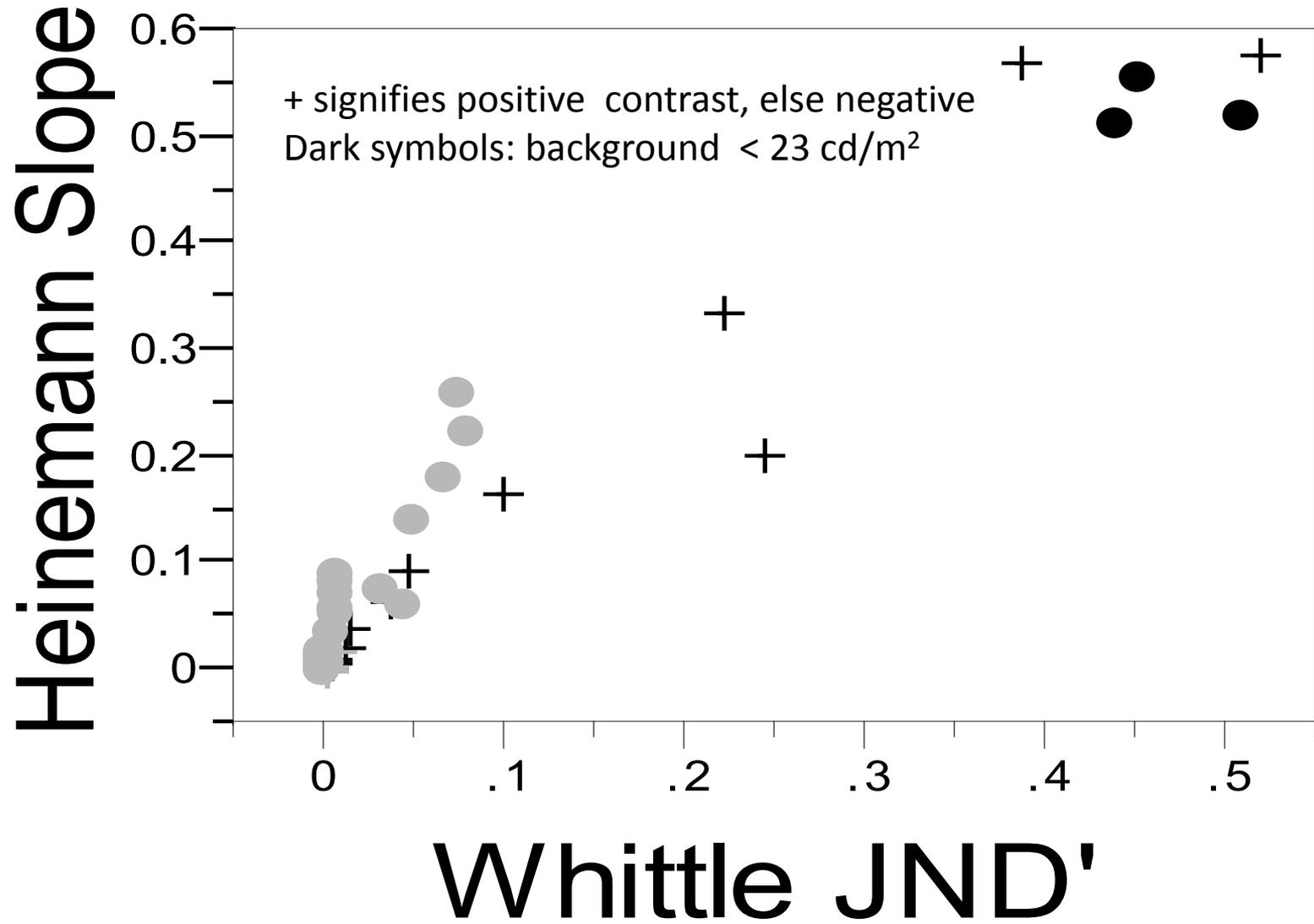
So,  $dJND/d(\log L) = L/\Delta L =$  claimed slope of JND in line 1 ,  
the slope of the gray scale.

But we really want  $dJND/dL = dJND/d(\log L)d(\log L)/dL = dJND/d(\log L)/L = 1/\Delta L$

How do these slopes compare with Whittle's slopes,  $JND'$ , at the same target and background luminances?...

# Plot of independent slope estimates

78 combinations of target and background luminances (over 5 log units)



# Conclusions

- Neutral (aka Gray, Brightness) scale is differentiable.
- *Slope of gray scale* is consistent, replicable, valid...
- Plotted “slopes” of gray scale correlate 0.966, N=78.
- The relationship has zero intercept and 45° line, i.e., slopes measured differently are plausibly identical.
- Slopes are valid for positive **&** negative contrasts.
- All this despite: 1) our two “slopes” were generated by very different experimental paradigms, by different labs 30 years apart; 2) slopes, in general, are notoriously twitchy.