Age- and field-size-parameterized calculations of physiologically significant XYZ colour-matching functions

Jan Henrik Wold[1] and Ivar Farup[2]
Buskerud and Vestfold University College[1], Gjøvik University College[2]
Drammen[1] , Gjøvik[2], Norway

In the CIE TC 1-36 technical report "Fundamental Chromaticity Diagram with Physiological Axes – Part I" [1], the 2° and 10° Modified CIE Colorimetric Observers are defined on the basis of cone fundamentals – i.e. the spectral sensitivity functions of the long-wave sensitive (L-), medium-wave sensitive (M-and short-wave sensitive (S-) cone mechanisms as measured in the corneal plane. The datasets adopted for this purpose are the 2° and the 10° cone fundamentals of Stockman *et al.* [2, 3], both of which have their psychophysical origin in the Stiles–Burch 1959 10° colour-matching data [4].

By particular reference to the procedure for derivation of the 2° data, the report of TC1-36 also provide a general guidance for age- and field-size-parameterized calculations of cone fundamentals. On the basis of the spectral absorbances of the cone photopigments (determined by Stockman *et al*. [3] by guidance of their own measurements of detection thresholds for dichromatic and normal observers under cone-isolating conditions), the fundamentals for different ages and field sizes are obtained by correcting for (1) the spectral absorption characteristics of the ocular media, (2) the spectral absorption characteristics of the macular pigment, and (3) the self-screening in the outer segment of the photoreceptors. The assumption made in these calculations are that (i) the spectral absorption in the ocular media depends solely on the age of the observer and (ii) the spectral absorption in the macular pigment and the self-screening in each type of photoreceptors depend on the visual angle only.

The method may be sketched as follows:

1. From the tabulations of the low-optical-density spectral absorbances $A_{i,0\,(cone)}(\lambda)$ (*cone* = *L*, *M*, *S*), the fraction of the incident light at the retinal level absorbed in the cones – i.e. the field-size-dependent cone absorptance spectra, $\alpha_{i\,,\,cone\,,\,fs}(\lambda)$ – are calculated by the equations

   $$\alpha_{i\,,\,cone\,,\,fs}(\lambda) = 1 - 10^{[-D_{\tau,\ max\ (cone),\,fs} \cdot A_{i,0\,(cone)}(\lambda)]} \quad (cone = L, M, S) \qquad (1)$$

   where $D_{\tau,\ max\,(cone),\,fs}$ (*cone* = *L*, *M*, *S*) are the field-size-dependent peak optical densities of the cone photopigments given as

   $$D_{\tau,\ max\,(L),\,fs} = 0.38 + 0.54\,e^{-fs/1{,}333} \qquad (2a)$$

   $$D_{\tau,\ max\,(M),\,fs} = 0.38 + 0.54\,e^{-fs/1{,}333} \qquad (2b)$$

   $$D_{\tau,\ max\,(S),\,fs} = 0.30 + 0.45\,e^{-fs/1{,}333}\ . \qquad (2c)$$

2. The fractions of the incident light at the corneal level absorbed in the cones – i.e. the age- and field-size-parameterized cone fundamentals, $\bar{l}_{fs,age}(\lambda)$, $\bar{m}_{fs,age}(\lambda)$ and $\bar{s}_{fs,age}(\lambda)$ – are given by the respective absorptance spectra multiplied by the transmittances of the pre-retinal media. Denoting by $D_{\tau,max,mac,fs}$ the field-size-dependent peak optical density of the macular pigment, $D_{\tau,rel,mac}(\lambda)$ the spectral relative optical density of the macular pigment, and $D_{\tau,ocul,age}(\lambda)$ the age-dependent optical density of the lens and other ocular media, the following equations apply:

$$\bar{l}_{fs,age}(\lambda) = \alpha_{i,L,fs}(\lambda) \cdot 10^{[-D_{\tau,max,mac,fs} \cdot D_{\tau,rel,mac}(\lambda) - D_{\tau,ocul,age}(\lambda)]} \quad (3a)$$

$$\bar{m}_{fs,age}(\lambda) = \alpha_{i,M,fs}(\lambda) \cdot 10^{[-D_{\tau,max,mac,fs} \cdot D_{\tau,rel,mac}(\lambda) - D_{\tau,ocul,age}(\lambda)]} \quad (3b)$$

$$\bar{s}_{fs,age}(\lambda) = \alpha_{i,S,fs}(\lambda) \cdot 10^{[-D_{\tau,max,mac,fs} \cdot D_{\tau,rel,mac}(\lambda) - D_{\tau,ocul,age}(\lambda)]} \quad (3c)$$

Here, the field-size-dependent peak optical density of the macular pigment, and the age-dependent optical density of the ocular media, are given by the equation

$$D_{\tau,max,mac,fs} = 0.485 \cdot e^{(-fs/6.132)} \quad (4)$$

$$D_{\tau,ocul,age}(\lambda) = \begin{cases} D1_{\tau,ocul}(\lambda)\,[1 - 0.02\,(age - 32)] + D2_{\tau,ocul}(\lambda) & \text{for } 20 \le age \le 60 \\ D1_{\tau,ocul}(\lambda)\,[1.56 - 0.0667\,(age - 60)] + D2_{\tau,ocul}(\lambda) & \text{for } age > 60 \end{cases}, \quad (5)$$

where $D1_{\tau,ocul}(\lambda)$ and $D2_{\tau,ocul}(\lambda)$ are tabulated functions representing, respectively, the portion affected by aging after age 20 and the portion stable after age 20.

In order to arrange for comparisons with the existing colorimetric standards [5], the TC1-36 [6] has decided to let the cone fundamentals for age 32 and visual angles of 2º and 10º be accompanied by XYZ representations determined in accordance with the general criteria imposed in the construction of the existing CIE standards [7,8]. For each representation to be unambiguously defined, some additional properties had to been implemented:

- the values of the color-matching function referring to the Z-primary are proportional to those of the S-cone fundamental.
- the difference between the chromaticity diagram's spectrum locus and the spectrum locus in the chromaticity diagram of the analogous CIE standard is, under the given constraints, a minimum according to a least RMS criterion.
- the spectral luminous efficiency function defining the alychne of the chromaticity diagram is taken as a linear combination of the L- and M-cone fundamentals, with relative weighting as proposed by Stockman *et al*. [9].

To follow up on the reports of TC1-36 [1,6], the CIE has established a technical committee, TC1-82, for which the terms of reference read:

1. Following on from CIE TR 170, to recommend a procedure for calculating XYZ-like colour matching functions from cone fundamentals, as a function of age and field size.
2. To deliver a computer program for the calculations.

With the age- and field-size parameterized cone fundamentals at hand, a main task of TC1-82 has been to decide upon a physiologically consistent concept for the age- and field-size-parameterized luminous efficiency function, $V_{F,\,age,\,fs}(\lambda)$, defining the alychne in the xy chromaticity diagram. Since, according to Eqs. (2a) and (2b), the peak optical densities of the L- and M-cone pigments are equal, the solution of the problem has been to let the corresponding quantum-based luminous efficiency function be defined as the linear combination of the age- and field-size-parameterized L- and M-cone fundamentals for which the relative weighting equals the weighting adopted by TC1-36 [6] (originally proposed by Stockman *et al*. [9]) for the particular cases of age 32 and visual angles of 2° and 10°. Accordingly, the quantum-based age- and field-size-parameterized luminous efficiency function, $V_{F,\,q,\,age,\,fs}(\lambda)$, is defined as

$$V_{F,q,\,fs,age}(\lambda) = \frac{1.89\,\bar{l}_{q,\,fs,age}(\lambda) + \bar{m}_{q,\,fs,age}(\lambda)}{[1.89\,\bar{l}_{q,\,fs,age}(\lambda) + \bar{m}_{q,\,fs,age}(\lambda)]_{\max}} \quad . \qquad (6)$$

To ensure that, for fixed age parameters, the spectrum loci are shifting smoothly as the field size parameter is gradually increased from 1° to 10°, the least-RMS optimisation involved in the determinations of the spectrum loci includes the derivation of field-size parameterised target loci that form a continuous transition between the spectrum loci of the CIE standard 2° and 10° observers (used as target loci in the particular cases of 2° and 10° field sizes). Examples of target loci for field sizes in the interval from 2° to 10° are shown in Figure 1.

A computer program for age-and field-size parameterized calculations of physiologically significant XYZ colorimetric systems is currently being developed within the TC1-82. In Figures 2 and 3 are shown examples of xy chromaticity diagrams and XYZ color-matching functions resulting from varying one parameter at a time.

The plan of the TC1-82 is to provide the computer program as a web application (written in Python), a MATLAB plug-in, and, if programming volunteers are found, an Excel macro.

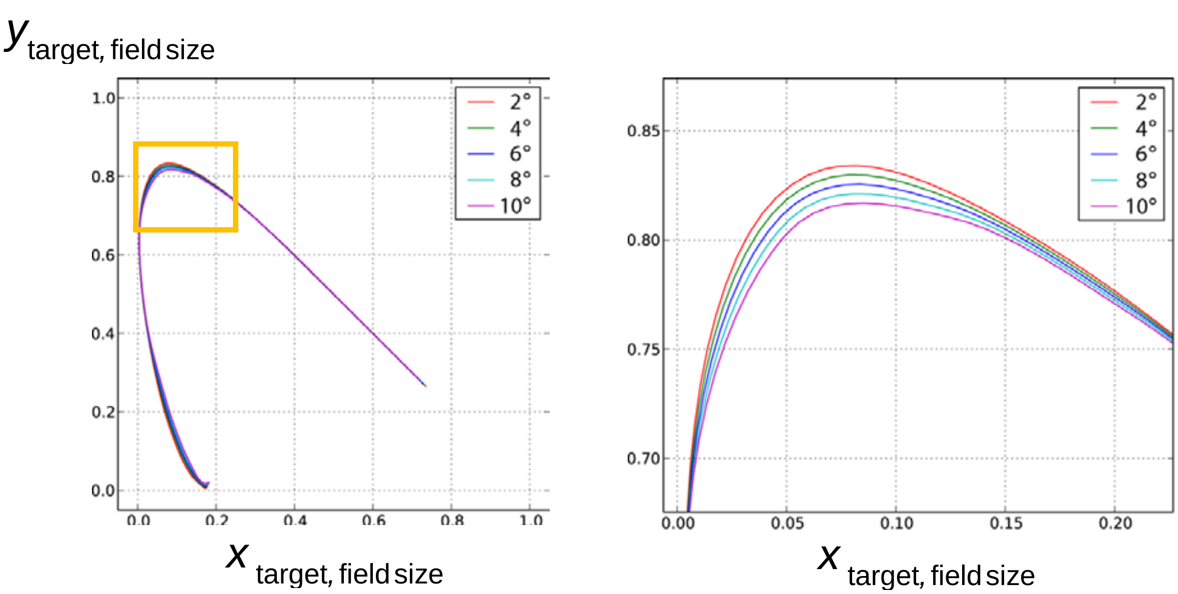

Figure 1: Target loci derived for determination of the spectrum loci at fixed age (32 years) and field sizes from 2° to 10°.

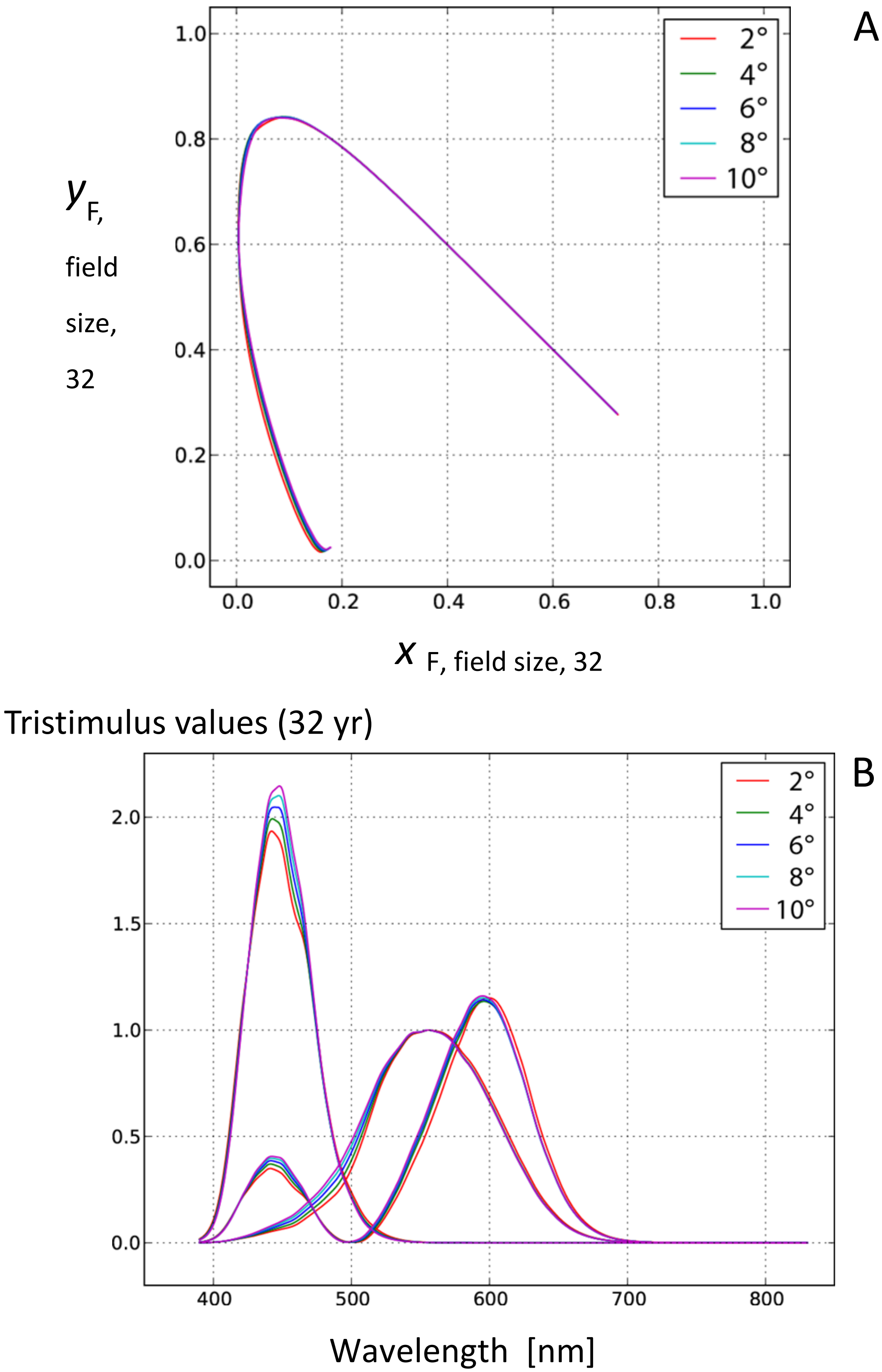

Figure 2: Physiologically significant xy chromaticity diagram (A) and XYZ

color-matching functions (B) for age 32 and field sizes from 2° to 10°.

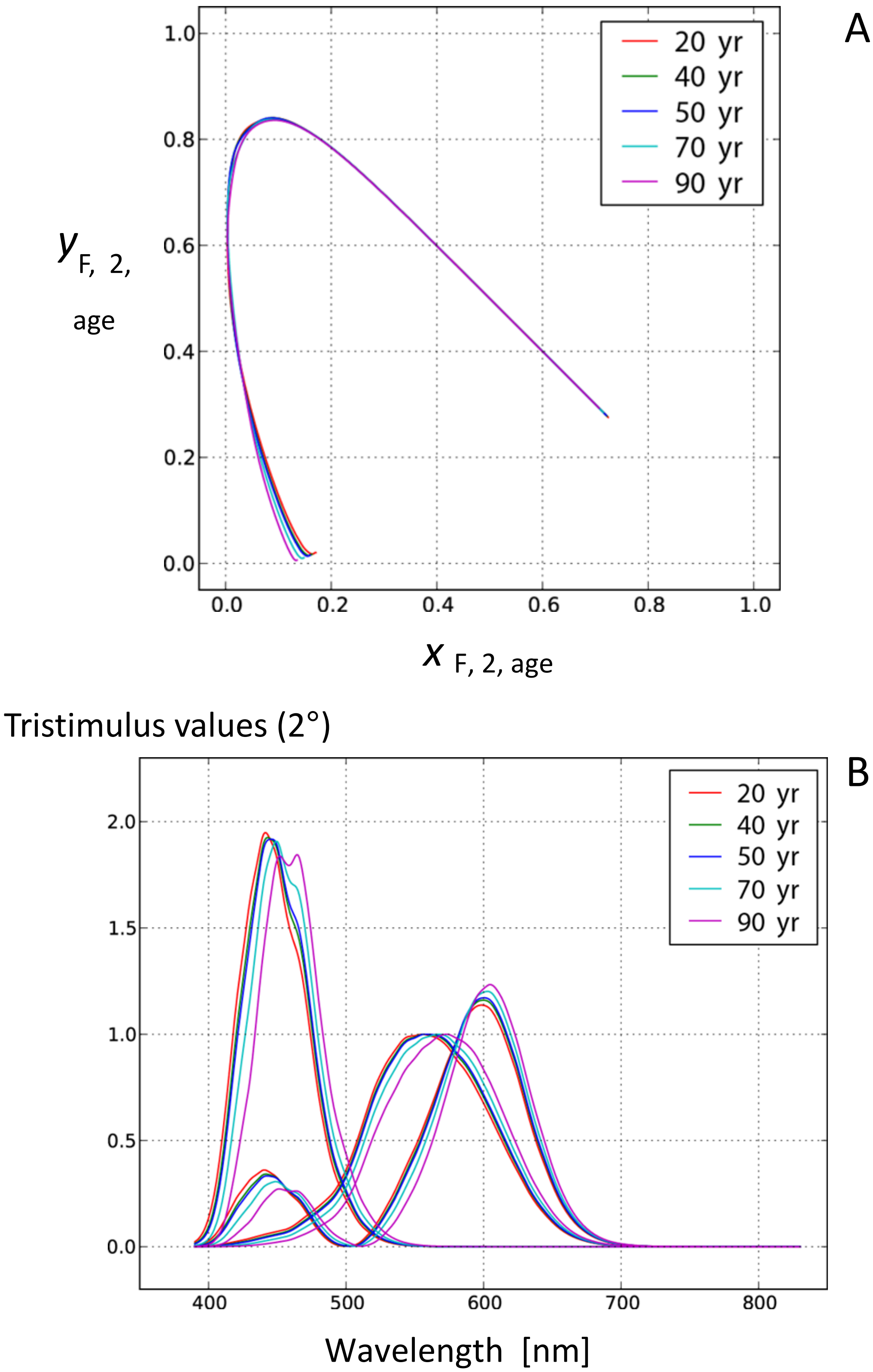

Figure 3: Physiologically significant xy chromaticity diagram (A) and XYZ color-matching functions (B) for field size 2° and ages from 20 to 90 years.

References

1. CIE. Fundamental chromaticity diagram with physiological axes – Part I. Publication CIE 170-1:2006. Vienna: CIE Central Bureau; 2006. 53 p.
2. Stockman A, Sharpe LT, Fach C. The spectral sensitivity of the human short-wavelength sensitive cones derived from thresholds and color matches. Vision Res 1999;39(17):2901–2927.
3. Stockman A, Sharpe LT. The spectral sensitivities of the middle- and long-wavelength-sensitive cones derived from measurements in observers of known genotype. Vision Res 2000;40(13):1711–1737.
4. Stiles WS, Burch JM. N.P.L. colour-matching investigation: Final report (1958). Opt Acta 1959;6(1):1–26.
5. ISO/CIE. CIE standard colorimetric observers. International standard ISO/CIE 10527:1991(E). Genève: International Organization for Standardization; 1991. 30 p.
6. CIE. Draft of TC1-36 Fundamental chromaticity diagram with physiological axes – Part II. (To be published by the CIE)
7. Smith T, Guild J. The C.I.E. colorimetric standards and their use. Trans Opt Soc 1931–32;33(3):73–134.
8. Judd DB. The 1931 I.C.I. standard observer and coordinate system for colorimetry. J Opt Soc Am 1933;23(10):359–374.
9. Stockman A, Jägle H, Pirzer M, Sharpe LT. The dependence of luminous efficiency on chromatic adaptation. Journal of Vision 2008;8(16):1,1–26, http://journalofvision.org/8/16/1/, doi:10.1167/8.16.1.

Author Biography

Jan Henrik Wold is assistant professor at the study program of Lighting Design at Buskerud and Vestfold University College, and has previously worked as assistant professor at Gjøvik University College and at The Oslo National Academy of the Arts. Wold is a member of the Norwegian National Committee of the CIE and sits as the Norwegian representative in CIE Div. 1, where he is currently serving as chair of TC 1-82 (The Calculation of Colour Matching Functions as a Function of Age and Field Size) and as a member of TC 1-36 (Fundamental Chromaticity Diagram with Physiologically Significant Axes), TC 1-85 (Update CIE Publication 15:2004 Colorimetry) and TC 1-94 (Visually Meaningful Spectral Luminous Efficiency Functions).