

Towards A Unified Method For Computing Tristimulus Values

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Introduction

Tristimulus values (TSV) form the basis of colorimetry. They are transformed to other colorimetric coordinates for industrial applications such as colour specification, colour quality control, colour difference evaluations, device characterization for cross-media colour reproduction and recipe formulation. Accurate calculation of TSVs is highly desired by industry for open environment communication.

The International Commission on Illumination (CIE) [1] originally defined the TSV in terms of the integrals:

$$V = \int_a^b W_v(\lambda)R(\lambda)d\lambda, \text{ with } V = X, Y \text{ and } Z \quad (1)$$

where

$$W_x(\lambda) = \kappa S(\lambda)\bar{x}(\lambda), \quad W_y(\lambda) = \kappa S(\lambda)\bar{y}(\lambda), \quad W_z(\lambda) = \kappa S(\lambda)\bar{z}(\lambda) \quad (2)$$

Here $W_v(\lambda)$ is the product of the relative spectral power distribution (SPD) of an illuminant, CIE colour matching functions (CMF) and the scaling factor. $R(\lambda)$ is the surface reflectance function (SRF) of the colour objects. (a,b) is the visible range of wavelengths with for example $a = 360\text{nm}$ and $b = 830\text{nm}$. Unfortunately, the integrands in equation (1) have no analytical expressions. CIE made a step further from 1986 by replacing the integrations by summations at 1nm steps:

$$V = \sum_{\lambda=a, \text{ step size } \Delta\lambda=1}^b W_v(\lambda)R(\lambda)\Delta\lambda, \text{ with } V = X, Y \text{ and } Z \quad (3)$$

However, the problems remain. The SPDs for the CIE illuminants D65 and A are available at 1nm intervals [2]. The CIE 1931 and 1964 CMFs are available at 1nm given by CIE [3]. Differently, not all the available spectrophotometers measure the SRF $R(\lambda)$ at 1nm intervals. Most of them measure at 5, 10, or even 20nm intervals. CIE [1] gives some guidance only for 5nm interval data, but has no precise recommendations for data with measurement intervals greater than 5nm. Thus, in practice, various approaches have been used for computing TSV, which can lead to a big discrepancy between two different methods from the same set of spectral data [4]. Hence there is a great demand from industry applications for a unified method for accurately computing the TSVs for the best agreement between laboratories, which led to the formation of CIE TC 1-71 on Tristimulus Integration during the 26th CIE Session in Beijing, 2007.

This talk will review all available methods including CIE recommendations. Comparison will be given and the talk is ended with discussions and recommendations.

Available Methods

The Direct Selection method: the earliest available method is the direct selection (DS) method, which sums all the products of the SPD, CMF and SRF at the same wavelengths as measured SRF. The method was also known as weighted ordinate method.

The CIE-R method: this is one of the CIE recommendations when the SRF is measured at intervals greater than 5 nm. The measured SRF is interpolated into 1nm and the 1nm summation is then used for computing TSV.

The ASTM tables 5 and 6 (T5 and T6): these two sets of weighting tables were standardized by ASTM Intl. in 1985 and 1995 respectively. Each set of tables consists of 36 weighting tables between 360nm and 780nm under 9 illuminants and two CIE standard colorimetric observers with measuring intervals being 10nm and 20nm respectively. The ASTM tables of tables 5 must be used with the measured SRF corrected by Stearns and Stearns correction formulae. While the ASTM tables 6 must be used with the measured SRF directly.

The Zero- and Second-Order Oleari Weighting Tables (OWT(0), OWT(2)): Oleari in 2000 gave a method for computing TSVs based on a local power (zero or second order) expansion. It was shown that the second order method is better than the ASTM T6 tables. His method computes TSVs from measured SRF. If there are many objects for computing the TSVs much computation work is repeated. For avoiding the repeat computations, recently Li et al derived the zero- and second-order weight tables based on Oleari's work. They also gave detailed computational procedures and worked example for implementation.

The Li-Luo-Rigg (LLR) method: this method was derived by Li et al in 2004 motivated by Venable's work. The LLR method is simple and computes optimum weights by solving three linear systems of equations with the same coefficient matrix being symmetric and positive definite tri-diagonal but different right hand side vectors.

The Li-Wang-Luo (LWL) method: this method was given by Li et al and Wang et al in 2005. Similar to the LLR method, the LWL method also computes optimum weights by solving three linear systems of equations with the same coefficient matrix being symmetric and positive definite tri-diagonal but different right hand side vectors. However, the coefficient matrix depends on the measuring wavelength interval, but the LLR's matrix does not. It was shown when the measuring wavelength interval goes to infinity, the two methods are the same. Hence it is expected that the two method perform similarly when the measuring wavelength interval is large and the two perform differently when it is small.

Performance Evaluation of Different Methods

In order to test different methods, the following evaluation procedure is used.

Evaluation Procedure

Step 1. Use the 1 nm standard weighting tables and 1 nm standard SRFs to compute the standard TSVs using the full visible wavelength range from 360 nm to 830 nm;

Step 2. Use the 1 nm standard SRF $R(\lambda)$ to derive the measured $\Delta\lambda$ interval data using equation (4) in a narrower range (a,b).

$$\hat{r}_j^{(\Delta\lambda)} = \int_a^b P_j(\lambda)R(\lambda)d\lambda \quad (4)$$

Step 3. Use a method or a method for deriving weighting table in the wavelength range (a,b) to compute TSVs using the measured reflectance values at Step 2. When the DS, ASTM T5, CIE-R and OWT(0) are used, the measured SRFs must be corrected using the Stearns and Stearns 3-term correction. All other methods/weighting tables use the measured SRFs directly.

Step 4. Compute the CIELAB colour differences between the TSVs obtained in Step 1 and those obtained in Step 3.

Note that the instrumental function $P(\lambda)$ in equation (4) has a triangular shape peaked at the nominal wavelength λ_j , and linearly decreasing to zero at λ_{j+1} and λ_{j-1} .

Note also that the 1nm standard SRF set used in the above procedure has 1096 SRFs between 360nm and 830nm at 1nm intervals. Six illuminants: D65, D50, A, FL2, FL7 and FL11 and two CIE standard CMFs are used for the evaluation. The 1nm SPDs and 1nm CMFs formed twelve 1nm standard weighting tables in the full range 360nm to 830nm. Thus, use the standard 1nm SRFs and 1nm weighting tables, the standard TSVs can be computed in Step 1. In Step 2, equation (4) is used for obtaining the measured SRF at $\Delta\lambda$ intervals between the narrower wavelength range (a, b). Currently, a=380nm and b=780nm are recommended by CIE. However, a=360nm and b=780nm are recommended by the ASTM [2,3]. The TSVs computed in Step 3 depend on the method used. If the TSVs obtained in Step 3 are the same as corresponding TSVs obtained in Step 1, the corresponding method is perfect and has no error in computing TSVs. However, in general it is not the case. The performance of each method is measured using CIELAB colour difference in Step 4. The smaller the colour difference, the better the corresponding method is. Six statistical measures are used for this evaluation. They are the average, median, 80 percentile, 95 percentile and maximum colour differences together the standard deviation of the colour differences.

Firstly, we consider the narrower range (360nm, 780nm) with $\Delta\lambda$ being 10nm and 10nm respectively in order to include the ASTM tables T5 and T6 for the comparison. It was found that when 10 nm weighting tables/methods are considered, numerical results show that the LWL method is the best for both continuous and fluorescent illuminants; ASTM table 6 is the second best under continuous illuminants and the LLR method is the second best under fluorescent illuminants; when 20 nm weighting tables/methods are considered, numerical results show that the LLR method is slightly better than the LWL method using continuous illuminants and the two methods are competitive using fluorescent illuminants. Overall, the LWL and LLR methods have similar performance and are better than all the other tested methods.

Secondly, we still consider the narrower range (360nm, 780nm), but with $\Delta\lambda$ being 2, 3, 4, 5, 6, and 7 nm respectively. In this case there are no standard ASTM tables. However, we have implemented ASTM T5 method. Hence ASTM T5 is included for this comparison. Comparison results also showed that in each case, the LWL method is the best.

Finally, comparison is made with the narrower range (380nm, 780nm), which is recommended by the CIE[1]. In this case, there are no standard ASTM tables and once again, we can compute ASTM T5 tables. Hence ASTM T5 is included for the comparison. Once again, similar results were found. When $\Delta\lambda$ is 20nm, the LRL and LWL methods are competitive and better than all other methods. When $\Delta\lambda=2,4,5,8$ and 10nm, the LWL is the best.

Discussions and Recommendations

Currently, CIE recommends that when $\Delta\lambda$ is smaller than or equal to 5, the DS method can be used for most practical applications not involving fluorescent materials. For $\Delta\lambda > 5$, the CIE has no unique recommendation, but gives some choices such as CIE-R method or the ASTM tables 5 and 6, or the LLR method can be used. According to the findings of this test, the LWL method should be used for any measured interval $\Delta\lambda > 1\text{nm}$. When $\Delta\lambda = 1\text{nm}$, the 1nm summation should be used. It is hoped that this talk or findings will promote uniformity of practice in the calculation of the CIE TSVs for practical applications. As noted by Billmeyer and Fairman [4] "there should be no reason for introduction of significant errors in this calculation process, even though other errors, such as those of specimen preparation or measurement, may be large in comparison".

References

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Author Biography

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